

SUPPLEMENT III
A NON-ARISTOTELIAN SYSTEM AND ITS NECESSITY FOR
RIGOUR IN MATHEMATICS AND PHYSICS*

BY ALFRED KORZYBSKI

We are here dealing with a concrete mathematical problem which is not trivial, but at the same time is solvable, and I cannot imagine that any mathematician can find the courage to elude its honest solution by means of a metaphysical dogma. (549)

HERMANN WEYL

I protest against the use of infinite magnitude as something completed which in mathematics is never permissible. Infinity is merely a *façon de parler*, the real meaning being a limit which certain ratios approach indefinitely near, while others are permitted to increase without restrictions. (74)

K. F. GAUSS

A very extensive literature shows that the problems of 'infinity' pervade human psycho-logical reactions, starting from the lowest stage of human development up to the present and that without some theory of 'infinity', modern mathematics would be impossible. Up to date, no satisfactory theory of infinity, on which all mathematicians could agree, has been produced. The results are rather bewildering because what appears to some prominent mathematicians as perfectly sound mathematics is evaluated by other equally prominent scientists as a 'mental' disease (Poincaré); or we find opinions that a large portion of mathematics is devoid of proof and has to be accepted on faith; or that some parts of mathematics must be treated as non-sense (Kronecker, Brouwer, Weyl.). 'There are eminent scholars on both sides and the chance of reaching an agreement within a finite period is practically excluded', says Brouwer, and certainly such a state of affairs does not allow us to have any satisfactory modern standards of proof and rigour; the last thing we should expect in mathematics.

The majority of those mathematicians who take interest in the soundness of their science seem to believe that the main difficulty centres around the validity of the 'law of excluded third' ('A is B. or not B') of the accepted, sharply two-valued, chrisippian form of A 'logic'. They disregard the fact that we are born, bred, educated, speak a language, live under conditions, institutions. , which still remain desperately A or even pre-aristotelian. If we attempt to reject one of the two-valued 'laws of thought' or postulates of the A-system but retain A or pre-aristotelian *elementalistic* 'psychologies', 'logic', and *s.r.*, no agreement in 'a finite period' can be expected, and the present mathematical chaos would continue.

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Among the more important schools we may distinguish roughly:¹

1) The logistic school represented by Peano, Russell, and Whitehead, who accept the chrisippian, two-valued, restricted form of the *el* 'logic' and so may be called the *chrisippian school*.

2) The axiomatic school, represented by Hilbert and his followers, which may be called the *aristotelian school*.

3) The 'intuitional' school represented by Brouwer and Weyl who question the 'law of excluded third', and so may be called the *non-chrisippian school*.

4) The Polish school of: (a) 'intuitional' formalism with Lukasiewicz, Tarski, Lesniewski as representatives, which may be called the *non-aristotelian school*. Lukasiewicz generalized the *A* 'logic' to three-valued 'logic' which covers modality. Lukasiewicz and Tarski finally produced a general many-valued 'logic' of which the two-valued represents only a limiting case. Lesniewski produced *Protothetic*, a still more general 'logical' system, by introducing variable 'funkctors',.* (b) The *restricted semantic* school represented by Chwistek and his pupils, which is characterized mostly by the semantic approach, and by paying special attention to the *number* of values, establishing the thesis that the older 'freedom from contradictions' depends on one-valued formulations, as discovered by Skarzenski and quoted by Chwistek. This school has already produced new foundations (still *elementalistic*) for 'logic' and mathematics, and leads to generalized arithmetics and analysis.

5) The average prevalent mathematical technician, who does not realize that he belongs to the numerically large class which may be called the 'Christian science' school of mathematics, which proceeds by faith and disregards entirely any problems of the epistemological foundations of their supposed 'scientific' activities.

It should be noticed that all existing mathematical schools accept implicitly, at least, *A elementalism* and do not challenge identity, a principle which happens to be invariably false to facts and which therefore should be entirely abolished.

The above classification suggests that, in spite of great achievements in the field of mathematical foundations, no school can expect to be convincing or accepted by other schools as long as we all flounder in the *A* and *el* ambiguities which prevent any possibility of agreement. It becomes obvious also that when a \bar{A} and *non-el* system is formulated it will necessitate a new paradox-free foundation for mathematics and so a new school of mathematics will arise which may be called:

6) The *general semantic, non-aristotelian, non-elementalistic* school of mathematics. It is premature to give the names of the leading pioneers in this field at present.

* At present Lukasiewicz and Tarski call their many-valued 'logic' non-chrisippian, but this name does not seem appropriate because these authors generalized both forms of the aristotelian 'logic' to a many-valued 'logic' of which the two-valued becomes only a limiting case. Thus it seems that their many-valued 'logic' is better described by the term *non-aristotelian*, yet still *elementalistic* 'logic'.

In a \bar{A} -system, the 'logical' problems of freedom from contradiction become also semantic problems of *one-valued meanings* made possible only under ∞ -valued, \bar{A} , *non-el* general semantics, and the recognition of the \bar{A} *multiordinality* of terms, . A \bar{A} -system introduces some fundamental innovations, such as completely rejecting identity, elementalism, and becomes based on *m.o* structure and order, and so ultimately becomes *non-el*. The A , (3+1)-dimensional *el*, (in the main) *intensional* system becomes a four-dimensional, *non-el*, (in the main) *extensional* system. In such a system we cannot use the formulations of *elementalistic* 'logics' and 'psychologies', but must have \bar{A} , *non-el* general semantics, which when generalized become an entirely general discipline applicable to all life, as well as to *generalized* mathematics. For the above reasons I shall use the word 'logic', in its *el* sense, with quotation marks; and use the term *general semantics* for a *non-el*, \bar{A} discipline corresponding to the *el*, A or \bar{A} 'logics'.

Investigations show that the primitive man (and the 'mentally' ill) use *one-valued semantics* which have left more or less marked traces in all of us, reflected even in science and mathematics. The elimination of these primitive traces clears the foundation for an adult civilization, a theory of sanity, and the elimination of the scientific and mathematical paradoxes.

To assume that because a many-valued 'logic' has been produced, all the problems of mathematical infinity, irrational numbers, continuity, mathematical induction, validity of mathematical proof, mathematical existence, have been solved, would be a mistake. The aim of the present paper is to analyse some of the fundamental complexities produced by the unconscious operation of the one-valued semantic *identification* concealed in the formulation of the 'law of identity', which have escaped notice until now, and which would make the application of a many-valued 'logic' or ∞ -valued semantics and agreement impossible. Here, as in the \bar{E} and \bar{N} systems, only the most general formulations help us to *discriminate* between the particular cases, and so to eliminate the undesirable traces of one-valued semantics by building a \bar{A} -system, of which the A and pre- A represent only particular cases.

Let me recall the 'philosophical grammar' of our language which we solemnly call the 'laws of thought', as given by Jevons:²

- 1) The law of identity. Whatever is, is.
- 2) The law of contradiction. Nothing can both be, and not be.
- 3) The law of excluded third. Everything must either be, or not be.

These 'laws' have different 'philosophical' interpretations which help very little and for my purpose it is enough to emphasize that: (1) The second 'law' represents a negative statement of the first, and the third represents a corollary of the former two; namely, no third possible between two contradictories. (2) The verb 'to be', or 'is', and 'identity' play a most fundamental role in these formulations. We should not be surprised to find that the investigation of these terms may give us a long sought solution. Such an investigation is very laborious and difficult. 'The complete attempt to deal with the term *is* would go to the form and matter of everything in existence, at least, if not to

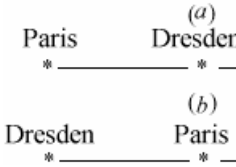
the possible form and matter of all that does not exist, but might. As far as it could be done, it would give the grand Cyclopaedia, and its yearly supplement would be the history of the human race for the time', said Augustus de Morgan in his *Formal Logic*, and this opinion I found fully justified.

So I must be brief, and state but roughly, that in the Indo-European languages the verb 'to be' has at least four entirely different uses: (1) as an auxiliary verb, 'Smith is coming'; (2) as the 'is' of predication, 'the apple is red'; (3) as the 'is' of 'existence', 'I am'; (4) as the 'is, of identity, 'the apple is a fruit'. The fact that four semantically entirely different words should have one sound and spelling appears as a genuine tragedy of the race; the more so since the discrimination between their uses is not always easy.

The researches of the present writer have shown that the problems involved are very complicated and cannot be solved except by a *joint study* of mathematics, mathematical foundations, history of mathematics, 'logic', 'psychology', anthropology, psychiatry, linguistics, epistemology, physics and its history, colloidal chemistry, physiology, and neurology; this study resulting in the discovery of a general semantic mechanism underlying human behaviour, many new interrelations and formulations, culminating in a \bar{A} -system. This semantic mechanism appears as a general psychophysiological mechanism based on four-dimensional order, present and abused in all of us, the primitive man, the infant, the 'mentally' ill, and the genius not excluded. It gives us an extremely simple means of training our *s.r.*, which can be applied even in elementary education.

The scientific problems involved are very extensive and can be dealt with only in a large volume. Here I am able to give only a very sketchy summary without empirical data, omitting niceties and technicalities.

If we consider an actual territory (a) say, Paris, Dresden, Warsaw, and build up a map (b) in which the order of these cities would be represented as Dresden, Paris, Warsaw; to travel by such a map would be misleading, wasteful of effort, . In case of emergencies, it might be seriously harmful, . We could say that such a map was 'not true'. , or that the map had a *structure not similar* to the territory, structure to be defined in terms of relations and multidimensional order. We should notice that:



A) A map may have a structure similar or dissimilar to the structure of the territory. (1)

B) Two similar structures have similar 'logical' characteristics. Thus, if in a correct map, Dresden is given as between Paris and Warsaw, a similar relation is found in the actual territory. (2)

C) A map *is not* the territory. (3)

D) An ideal map would contain the map of the map, the map of the map of the map. , endlessly. This characteristic was first discovered by Royce. We may call it self-reflexiveness. (4)

Languages share with the map the above four characteristics.

A) Languages have structure, thus we may have languages of *elementalistic* structure such as ‘space’ and ‘time’, ‘observer’ and ‘observed’, ‘body’ and ‘soul’, ‘senses’ and ‘mind’, ‘intellect’ and ‘emotions’, ‘thinking’ and ‘feeling’, ‘thought’ and ‘intuition’ ., which allow verbal division or separation. Or we may have languages of *non-elementalistic structure* such as, ‘space-time’, the new quantum languages, ‘time-binding’, ‘different order abstractions’, ‘semantic reactions’ ., which do not involve verbal division or separation.; also mathematical languages of ‘order’, ‘relation’, ‘structure’, ‘function’, ‘variable’, ‘invariant’, ‘difference’, ‘addition’, ‘division’ ., which apply to ‘senses’ and ‘mind’, that is, can be ‘seen’ and ‘thought of’ . (5)

B) If we use languages of a structure non-similar to the world and our nervous system, our verbal predictions are not verified empirically, we cannot be ‘rational’ or adjusted, . We would have to copy the animals in their wasteful and painful ‘trial and error’ performances, as we have done all through human history. In science we would be handicapped by semantic blockages, lack of creativeness, lack of understanding, lack of vision, disturbed by inconsistencies, paradoxes, . (6)

C) Words *are not* the things they represent. (7)

D) Language also has self-reflexive characteristics. We use language to speak about language, which fact introduces serious verbal and semantic difficulties, solved by the theory of *multiordinality*. (8)

The above unusually simple considerations lead to unexpectedly far-reaching consequences.

A) From (7)—it follows that the objective levels which include the events, ordinary objects, objective actions, processes, immediate feelings, ‘instincts’, ‘ideas’, *s.r* in general. , represent un-speakable levels, *are not words*. (9)

B) From (9)—that the use of the ‘is’ of *identity*, as applied to objective, un-speakable levels, appears invariably structurally false to facts and must be entirely abandoned. Whatever we might *say* a happening ‘is’, *it is not*. (10)

C) From (10)—*structure* appears as the only possible link between the objective, un-speakable, and the verbal levels. (11)

D) From (11)—the only possible ‘content of knowledge’ becomes exclusively *structural*. (12)

E) From (12)—the only aim of ‘knowledge’ and science appears as the empirical search for, and verbal formulation of, structure. (13)

F) The only method for acquiring ‘knowledge’ is found in an *empirical* investigation of the potentially unknown structure of the world, ourselves included, only afterwards adjusting the structure of languages so that they would be similar, and so of maximum usefulness; instead of the delusional

reversed order of ascribing to the world the structure of an inherited primitive language. (14)

G) The investigation of the potentially known structure of languages in which we predict and then verify the predictions empirically, appears as an important method for the discovery of the structure of the world. (15)

H) Investigations disclose that all *A, el* languages and disciplines built on them (older 'psychologies', 'logics'. , and, based on them, economics, sociology, politics, 'ethics'. , reflected in turn in our institutions, systems. .) *are not* structurally similar to the world and our nervous system, as they verbally divide what empirically cannot be divided. Under such conditions neither a higher grade civilization, nor general sanity, nor paradox-free science and mathematics are possible. In *el* languages, our verbal predictions are not verified empirically, and not being able to foresee we must proceed by animalistic 'trial and error'. (16)

I) Mathematics appears as a very limited but the only language in existence, in the main similar in structure to the world around us *and* the nervous system. (17)

J) From the study of mathematics, mathematical physics, and physics, we learn, and will continue to learn, the fundamentals of *m.o* structure. It is no mystery that all chemistry has become a branch of physics, all physics can be made a branch of geometry, all geometry a part of analysis, and all analysis a part of general semantics. The present work shows that the analysis *of all human problems* of daily life or science becomes dependent on *general semantics* which on the verbal levels becomes generalized mathematics. Thus mathematics, mathematical physics, and physics become the most important disciplines from which we learn most about *structure*,—the only 'content of knowledge'. (18)

K) The older *el* 'psychologies' and 'logics' for their maximum usefulness must be transformed into unified *non-el* psycho-logics and general semantics, possible only after studying all forms of human behaviour, mathematics included. (19)

L) The study of mathematics as a form of human behaviour, appears necessary prior to the possibility of formulating any laws of semantics. (20)

M) The problem of mathematical foundations do not belong to mathematics but to psycho-logics which would not disregard anthropology, and would not be vitiated by our persistence in the use of structurally inappropriate *el* 'psychologies', 'logics', and an innocence of mathematics. (21)

N) The 'intuitional' and the 'intuitional' formalist schools of mathematics must be considered as a legitimate, yet not properly formulated, protest against the older elementalism. (22)

O) The general semantic school will represent the *non-el* and \bar{A} school of mathematics. (23)

P) The present crisis of mathematics ultimately depends on the meanings and use of a few terms such as 'all', 'there is', 'infinite'. , which solution depends on a *non-el* theory of meanings, which ultimately can be solved by transforming

what might be called the (3 +1)-dimensional *el*, *A*-system, which divides ‘space’ and ‘time’. , (an attitude which is carried all through the system), into a four-dimensional *non-el*, \bar{A} -system (an attitude which is also carried all through the system). (24)

Q) From (8)—it follows that statements about statements represent results of new neurological processes, that their content varies, and that we must *discriminate* and *not identify* these different meanings. In other words, only through consciousness of abstracting which represents the most general *s.r* of discrimination, or the elimination of identification, can we assign single values to words which have an essentially many-valued character. Identification confuses these many meanings into one. (25)

R) We must differentiate between descriptive and inferential words and phrases, and never use inferential terms as descriptive, without realizing that we are doing so. (26)

S) Certain words or phrases used to speak about languages, such as ‘all statements’, ‘proposition about all propositions’. , lead to self-contradictions. We cannot speak about ‘all’ propositions without some limitations, if we proceed introducing new propositions. Even St. Paul felt the necessity for limiting the values of ‘all’. * We are compelled to introduce some equivalents to the biblical ‘illegitimate totalities’ or the theory of types of Russell. (27)

T) Analysis finds that certain of the most important terms we use; such as, ‘yes’, ‘no’, ‘true’, ‘false’, ‘all’, ‘fact’, ‘reality’, ‘existence’, ‘definition’, ‘relation’, ‘structure’, ‘order’, ‘number’, ‘is’, ‘has’, ‘there is’, ‘variable’, ‘infinite’, ‘abstraction’, ‘property’, ‘meaning’, ‘value’, ‘love’, ‘hate’, ‘knowing’, ‘doubt’. . . , may apply to all verbal levels and in each particular case may have a different content or meanings and so *in general no single content or meaning*. I call such terms *multiordinal terms (m.o)*. The definition of such terms is always given in other *m.o* terms preserving their fundamental multiordinality. In other words, a *m.o* term represents a many-valued term. If the many values are identified, or disregarded, or confused, we treat a fundamentally many-valued term as one-valued, and we must have every kind of paradox through such an identification. All known paradoxes in mathematics and life can be manufactured by the disregard of this fundamental multiordinality. Vice versa, by formulating the general semantic problem of multiordinality we gain means to discriminate between the many meanings and so assign a single meaning in a given context. A *m.o* term represents a variable in general, and becomes constant or one-valued in a given context, its value being given by that context. Here we find the main importance of the semantic fact established by Skarzenski,** that the ‘logical’ freedom from contradiction becomes a semantic

* Professor Cassius J. Keyser drew my attention to a passage in the first letter of St. Paul to the Corinthians, Chapter 15, line 27. ‘For he hath put *all* things under his feet. But when he saith *all* things are put under him, it is manifest that he is *excepted*, which did put *all* things under him.’ Italics are mine.

** Quoted by Chwistek in his *Neue Grundlagen der Logik und Mathematik*.

problem of one-value. But for application we must have a four-dimensional, *non-el*, \bar{A} extensional system, based on structure. , and the complete elimination of identity. (28)

U) That the disregard of multiordinality, orders of abstractions, may lead to identification and therefore false evaluation resulting in disagreement and maladjustment. (29)

V) From (25-29)—it follows that identification or confusion of higher order abstractions must be eliminated. Because of (7, 9, 10, 25-29)—*all identification must be eliminated.* (30)

W) The elimination of identification on all levels, or a complete and unconscious discrimination between different orders of abstractions, including as a special important case the multiordinality of terms, results in general consciousness of abstracting which in turn, solves the paradoxes of life and mathematics and leads to *generalized* mathematics along the lines suggested by Chwistek. (31)

X) The realization of the inherent multiordinality of some of the most important terms we have, gives us an enormous flexibility of language. It makes the number of our words indefinitely great. When both the writer and the reader recognize this multiordinality, and look for the meaning in the context and discriminate between the orders of abstractions, indicated by the context, confusion becomes impossible. (32)

Y) The test for multiordinality is simple. We take any statement and test it to see whether a given term applies to it. Then we make a statement about this statement and again test if this term applies to the new higher order statement. If it does, the given term must be considered multiordinal, because this procedure may be repeated indefinitely. (33)

Z) *The complete elimination of identification* does not allow us to use the term 'is' of identity, and so we must use operational, functional, actional, behaviouristic. , languages, requiring new attitudes and new *s.r.*, impossible without the formulation of \bar{A} -system. (34)

Z₁) The *s.r* of those who produced the general theory of relativity, the unified field theory, the new quantum mechanics, the new revision of the foundation of mathematics. , depend on new \bar{A} , *non-el*, and non-identity, operational, actional. , attitudes. (35)

Z₂) As the \bar{A} -system is based on the general elimination of the 'is' of identity, or on 'is not', it is impossible to reject these premises without producing impossible data, and a theory of agreement 'in a finite period' then becomes a possibility. (36)

Z₃) The old 'unknowable' becomes abolished and limited to the simple and natural fact that the objective levels *are not* words. (37)

Observation and experience, scientific and otherwise, show that in nature we find a definite order, which establishes a *natural order*; namely, that the sub-microscopic process, called the event or the scientific object, came first; only later abstracting organisms happened and objects which represent the results of abstracting by amoebas or men, came next. In the process of evolution

we find object first, label next. Descriptions first, inferences next. The above *natural order* establishes also a *natural order of evaluation*. Proper evaluation becomes the foundation for survival, *non-el s.r.*; the more so since evaluation requires asymmetrical relations of ‘more’ or ‘less’. , impossible to handle properly in an *A*-system. Thus the most important level is represented by the sub-microscopic processes. What the organism needs is not the three-dimensional shadow of a four-dimensional event, not the abstraction of low order produced by our nervous systems, called the object, but the sub-microscopic dynamic processes without which the desired end-results would not happen. The animal, the primitive, the infant, the ignorant man identify the two; live in a delusional world. Similarly the objective levels are more important than the verbal levels, and descriptions are more important than inferences. If we *identify* any orders while the natural order is established by the asymmetrical relation of ‘more’, the semantic process of evaluation is *reversed* and appears pathological in different degrees. If $a > b$ and we make them delusionally equal in value (identify), then, in the false-to-fact relation $a = b$, we have either over-evaluated the right-hand side or under-evaluated the left-hand side; in both cases reversing the natural order of evaluation. It is important to notice that by basing our *s.r* on a *natural order* of evaluation, general semantics become a generalized science of order and values; a very secure guide in life, indispensable for sanity, as experiments have shown, and include also generalized mathematics.

Another very serious mechanism of identification is found in language.

A) Thus we have only *one name*, say ‘apple’ for the: (a) un-speakable, un-eatable event or scientific process; (b) the un-speakable but eatable abstraction of low order, the object; (c) the un-speakable and un-eatable ‘mental’ picture, or higher order abstraction, on semantic levels; (d) and for a definition on verbal levels.

(38)

B) The multiordinality of terms was not discovered until 1925 and is still generally unknown. It presents a serious difficulty facilitating, perhaps even necessitating, identification unless prevented by special formulations and training. Multiordinal terms sound and look alike on all levels; experience has shown how easy it is to confuse their orders and identify the many values into one.

(39)

C) The differentiation between descriptions and inferences, and particularly between descriptive and inferential words as such, is also novel, and was, until the present *A*-system was formulated, largely disregarded, which again led to identifications and false evaluations.

(40)

Investigations show, that in all known primitive peoples and in the ‘mentally’ ill, we find literal identification of different orders of abstractions, which accounts for these semantic states. Even their ‘perceptions’ are different from those of the so-called ‘normal’, ‘civilized’ man, because higher order abstractions are projected and identified with lower order abstractions. They identify or ascribe one value to essentially many-valued different orders of

abstractions and so become impervious to contradictions with 'reality' and impervious also to higher order experience.

The infant, and the rest of us, identify a great deal because of the reasons given above. Investigations show that most of human difficulties, public, private, or *scientific* are due to this *A s.r.*, which accounts for the infantile state of our commercial so-called civilization. Identification abolishes the natural order of evaluation, but so does also an unconscious assumption of an 'infinite velocity' of a process. The *A* trilogy involved some fanciful 'infinity' assumptions. Thus in the *A*-system the velocity of nerve currents, which is known to be 126 metres per second in the human nervous system, is at present assumed unconsciously as 'infinite', made evident by the elementalism of 'intellect' or 'emotions'. , as something 'by themselves' and detached. In the *E*-system the length of a line, the space constant, and the natural unit of length were assumed 'infinite'. In the *N*-system the velocity of light, known to be finite, was unconsciously assumed to be 'infinite'. In the *A* trilogy these unjustified or meaningless 'infinities' have been eliminated. 'Infinite velocity' of a process has no *meaning*. It represents only a play upon symbols. Velocity is defined as $v=s/t$. If we assume $t=0$ and write $v=s/0=\infty$, this 'velocity' lacks one of the fundamental factors of its definition; namely, t , and so such an expression ceases to define anything at all and has no meaning, although it may be a symbol for a semantic disturbance. But the results of such delusional *s.r* are far reaching, no matter how mild they might be in degree. In a process propagated with 'infinite, velocity there would be no transition or delay in action, and therefore such a process would *not be ordered*. Vice versa, the disregard of order in our observations must introduce some mythological 'infinities' somewhere. So we see that the semantic process of identification is intimately connected with 'infinity' assumptions, both *abolishing order*. Training in natural order trains *s.r* away from delusional evaluation, abolishes pathological identification of different degrees and fanciful 'infinities'.

Thus we see that the problems of mathematical 'infinity' are extremely complex and involve many fundamental considerations never analysed before in connection with the semantic process of identification. Once these problems are analysed and formulated from a *A, non-el*, structural point of view the problems of 'excluded third' become secondary in importance, easily managed under the creative freedom of the coveted 'consciousness of abstracting'.

Let me recall for continuity, that the mathematicians recognize at present, two kinds of 'infinities'. One with which we are familiar from our school days, symbolized by ∞ , Cantor calls 'potential' infinity and defines as a *variable finite*, the misunderstanding of which introduces paradoxes even in high schools; the other, the 'actual' infinity, which introduces paradoxes in universities. All these paradoxes are due, as the present enquiry shows, to fundamental fallacies in connection with semantic processes of identification which we learn at home and in elementary schools.

The process of identification of different orders of abstractions may be due to pathological conditions, to ignorance, to 'thoughtlessness', to lack of

observation, to unconscious false assumptions, to hastiness, to superficiality, to habits of speech, to the structure of language used, . In fact, under the *A*-system it is practically impossible to avoid it, as we can witness it in such a comparatively advanced field as mathematics. The label 'identification' is applied to the semantic process of wrong evaluation going on inside of our skins on the un-speakable objective levels, when we are not aware of the differences between different orders of abstractions. When making it conscious, we may speak of the confusion of the orders of abstractions. To make such a process conscious, we must train in the differentiation or discrimination between different orders of abstractions, and distinguish the different orders by actually learning how to order them. Such training results in general *consciousness of abstracting* which is not inborn, nor fully acquired, even in university training, but which requires *special* training. Experiments in this field are extremely encouraging; in a number of cases, pathological individuals have become 'normal' and the 'unchangeable' human nature has been actually changed. *Infantile reactions in adults are abolished*, and this training becomes a general and simple method for prevention of future semantic disturbances of false evaluation which must result in maladjustment.

To stop identification we must discriminate or differentiate to the limit between what appears always as four-dimensional, absolute individual stages of processes and situations on all levels, verbal included. Let us follow briefly such an actual performance. If we realize (7)—we accept (11-15)—and on *structural* grounds reject the elementalism of the *A* trilogy as expressed in its 'psychology', 'logic', the division of 'space' and 'time', . We accept the non-elementalism of the \bar{A} trilogy as expressed in the new terms in the present work and accept also 'space-time', . The difference is very serious in all fields, when carried consistently all through the system. As we actually deal with four-dimensional dynamic processes which must be considered continually different, and with world conditions changing also continually, statements about such structural conditions, in an extensional sense, must be considered as involving variables, generating *propositional functions*, doctrinal or system-functions, funktors, . But propositional functions, which involve variables, are neither 'true' nor 'false', but ambiguous, and to have a proposition we must assign a value to the variable by *at least* permanently, in principle, assigning a date to it. We must also introduce, in principle, and as a semantic attitude, numerical subscripts to our words. Thus 'apple' in the *A*-system represents a *name* attached to an *intensional definition*, and space-time considerations do not enter. The term is applied to a definition which might be considered as one-valued and permanent. Now obviously such a language and *s.r* are *structurally non-similar* to the world and our nervous system.

If we try to identify a name for a definition, implying permanence, with the objective level which is made up of absolute individuals, and represents ever-changing processes, we must live in a delusional world in which we should expect every kind of paradoxes and psycho-logical shocks.

In a \bar{A} -system, for structural reasons, we must retain the general implications of the term 'apple', so we retain the word. We must make our language extensional in principle, and the name 'apple' an *individual name*, by calling it 'apple₁', 'apple₂', . . . The combination of letters 'a-p-p-l-e' implying similarities, the subscripts 1, 2, . . . implying individual differences, which automatically prevent identification. But this is not enough. Our 'apple₁' represents a name applied to an object *and* a process; its *meaning* becomes only one-valued when we assign to it at least a definite date. Thus the objective 'apple₁ (Dec. 1, 1931.)' may be a very appetizing affair, and 'apple₁ (Jan. 1, 1932.)' an un-edible wet splash. It should be noticed that the fundamental difference between the A and \bar{A} systems turns out to be a difference of semantic attitudes. The scientific facts are not changed. The 'apple' of 'Adam' or our own did not differ in essential characteristics under discussion. In both the A and \bar{A} -systems we actually deal, in principle, with *many-valued processes*. The important problem is to adjust the structure of our verbal processes to the structure of the world; hence a \bar{A} -system must be made extensional, *non-el*, four-dimensional, . . . Here once more, as in general semantics, the ascribing of one value (or at least limited to a small range of values in practice), in a given situation (context), eliminates paradoxes and contradictions on the older 'logical' grounds. We should notice that the multiordinal terms must be considered as names for many-valued *s.r.*, depending upon the order of abstractions; hence the name *multiordinal*. Names for happenings on the objective levels apply to many-valued processes but should not be considered multiordinal. All the psycho-logics of the differential calculus, 'space-time,' enter here, yet the whole field is covered semantically if we entirely abandon the 'is' of identity. Instead of training in 'allness' and 'isness'—'this *is* this', we shall train in non-allness, and non-isness—'this *is not* this', in connection with a special diagram called the Structural Differential.

Experience and experiments show that the above seems essential for sanity. It is interesting to notice that mathematics has produced a language similar in structure to the human nervous system. Roughly the central part of the brain which we call the thalamus is directly connected with the dynamic world through our 'senses' and with those semantic manifestations which we usually call 'affective', 'emotions'. . . all of which manifest themselves as dynamic. The cortex which gives us the static verbal reactions and definitions, is not connected with the outside world directly but receives all impulses through the thalamus. On semantic levels the thalamus can only deal with dynamic material, the cortex with static. Obviously for the optimum working of the human nervous system, which represents a cyclic chain, where the lower centres supply the material for the higher centres and the higher centres should influence the lower, we must have means to translate the static into dynamic and the dynamic into static; a method supplied *exclusively* by mathematics.

With the above considerations we must discriminate between our semantic capacities for *infinite divisibility* of finites, and for the *generation* of infinite postulated processes which by definition *cannot be exhausted*. If we use a three

dimensional A language and apply such an 'all' to such an infinite process then we simply produce a self-contradiction. If we apply to such a semantic process a four-dimensional 'all with a date', then we have arrested, for the 'time' being, the process, or taken a static cross section of the infinite process at that date; but then we deal with a finite. Once we are constantly conscious of abstracting in different orders, these subtle differences become quite clear and the solutions of the problems of infinity follow a similar path as the older problems of the 'infinitesimal,' which also was self-contradictory, unnecessary for mathematics. When treated as a *variable finite* it was satisfactory and sufficient, and has proven to be a most creative notion in mathematics. In the problems of the irrational, continuity, , similar subtle identifications or non-discriminations of *el*, \bar{A} , three-dimensional terms with \bar{A} , *non-el*, four-dimensional terms occur, which once eliminated, clear up not only the paradoxes, but some self-contradictory, often unconscious, postulates of some parts of mathematics.

Lack of space does not allow me to go into further details, except to suggest how some subtle *discriminations* may help to eliminate identification. In my \bar{A} -system the differentiation between orders of abstractions on physiological grounds, the introduction of multiordinality of terms, four-dimensional considerations, , as a structural necessity for all languages, makes the theory of types unnecessary.

For a better understanding of the present work we must at least differentiate:

- A) Between numerical experience and mathematics.
- B) Between languages with content and languages without content.
- C) Between creative building of verbal schemes which, for the sake of generality, have no content, called pure mathematics, and the application of these schemes to actual problems, with content, called applied mathematics.
- D) Between the contentless mathematics and the investigation of the foundation of mathematics which represents the investigation of the *s.r* of mathematicians and belongs to a future *non-el* psycho-logics with content.
- E) Between different forms of complex adjustment which we have in common with the primitive man, and even the higher animals, and reasoning which starts with conscious observations, passing to descriptions and inferences, .
- F) Between the dynamic process of relating ('thinking') on the unspeakable semantic levels, and the verbal expression of 'relations'.
- G) Between the use of negative terms, disagreement, contradiction; and self-contradiction. In a \bar{A} -system contradictions take the form of self-contradictions.
- H) Between *el* 'logics' expressed in terms of 'true', 'false', and modality, and the investigations of one-, two-, three-, and ∞ -valued *s.r*, which become a *general theory of values*, and which may some day include all human interests.
- I) Between the inherent circularity of 'human knowledge', which must start with sets of undefined terms, and so start with some knowledge, and circular definitions or explanations which define or explain nothing.

This list of suggestions is not exhaustive, and in principle appears as inexhaustible. I selected only a few topics of immediate need.

We should also notice that because on objective levels we deal structurally with absolutely individual stages of processes and situations and by necessity we speak in higher order abstractions and generalities and use many multiordinal terms (without the use of which no speaking is possible), so *any positive statement* about the objective levels must be only probable in different degrees which introduces a fundamental and entirely *general \bar{A} principle of uncertainty*. Heisenberg's restricted principle in physics appears only as a special case. For structural reasons we must preserve determinism but because of (11-15)— the older two-valued determinism must be reformulated into the ∞ -valued determinism of the maximum probability. The einsteinian introduction of non-elementalism in physics has resulted in the automatic elimination of some semantic blockages in the younger physicists. Some of the semantic results and triumphs of science, besides the new quantum mechanics, can be found in the latest (free from identification of the term 'time' with some objectivity) new entropy of Tolman.³

To sum up, we find that although the primitive man or the 'mentally' ill may have some reactions of orientation, or capacity for relating, which we have in common with the higher animals, yet these do not involve 'reasoning' in the sense defined before. Thus a boxer, football player, ., does a great deal of reflexive relating and wins his match, but this cannot be considered as reasoning in the strict sense as used in \bar{A} -system. If we attempt to *discuss* something with a primitive or 'mentally' ill individual and write down his processes of relating, we would have to conclude that he uses *one-valued semantics of identification* of many values into one, or a semantics of inclusion by which 'everything is everything else'. The 'law of contradiction', or any 'excluded' 'third', or '*n*-th', practically never appears in our sense, yet it is complicated by the use of positive and *negative* terms, to which any meanings connected with some identifications of higher orders may be ascribed. Although his prevailing semantic processes appear as a complete and *literal identification*, yet because of the general orienting and relating capacities of organisms and the character of terms used, it would not be easy or profitable to attempt an *el* formulation of his 'laws of thought'. But a semantic formulation, as given above, is very instructive and comparatively simple.

Our existing *el* 'logic', besides the two-valued type of formulation, involves many different 'philosophical' elucidations, which instead of clarifying the status of 'logic', in general, tends only to conceal the important issues involved in *non-elementalism*. The role that identification plays in a given individual appears always as a deciding factor in his adjustment. Unfortunately, at present, the sinister identification is not counteracted but fostered or even induced by the structure of the languages we use, different mythologies, ., and our whole educational, economic, social, ., systems.

The two-valued *A, el*, three-dimensional 'logic' does not apply to the world of events, to the objective levels, ., and, for the reasons already explained,

does not apply to the study of the foundations of mathematics. It applies to a large extent to contentless technical mathematics, including so-called 'formal logic' of that system.

Formalism *when free* from identification becomes a unique comparative tool in search for structure; formalism *with* identification of different orders of abstractions, a symptom of semantic disturbances, often of a morbid character. It should be realized that we may have one-, two-, three-, many, and ∞ -valued orientations, which with the exception of one-valued, we should utilize when conditions warrant a particular use in a particular case. Thus in mathematics, for the sake of having mathematics as a standard of evaluation, *we select* a sharply two-valued orientation by which in the old language 'A is B or not B', *to allow sharp* statements that for instance, $1+1=2$. If we would deliberately postulate that $1+1$ may sometimes be equal to 2 and sometimes not equal to 2, we would have forms of representation which would apply perhaps more readily to science and life, but mathematics as such would be impossible, and we would be deprived of this sharp tool for evaluation.

It is interesting to notice that mathematicians, by the use of two-valued *semantics*, (not 'logic', because an *el* discipline cannot be 'lived through' at all by non-heavily pathological individuals), have produced the most important disciplines. Thus we have, for instance, the theory of 'variance' (the theory of function), the theory of invariance, the differential calculus, the 1, 2, 3, 4, and n -dimensional systems, and a host of other verbal structures similar not only to the world, but to the human nervous system. These results give us means not only to enlarge our mastery of the external world, but when generalized into a *non-el*, \bar{A} -system, give us the means for the mastery of the inner world, leading toward sanity.

It is amusing to discover, in the twentieth century, that the quarrels between two lovers, two mathematicians, two nations, two economic systems, usually assumed insoluble in a 'finite period' should exhibit one mechanism—the semantic mechanism of identification—the discovery of which makes universal agreement possible, in mathematics and in life.