CHAPTER XXXVIII

ON THE 'WORLD' OF MINKOWSKI

> Moreover, the really fundamental things have a way of appearing to be simple once they have been stated by a genius, who was in this case Minkowski.
>
> (431) G. Y. RAINICH

We have already freely used the structural term 'dimension' and only hinted at its meanings. Before we approach the Minkowski world we must summarize roughly what for our purpose we should know about dimensions.

There is nothing mysterious about the term 'dimension'. First of all, the dimensionality of a manifold is not inherent in the manifold as such. It is a characteristic of *order* and so of structure. A manifold can be ordered in different ways, so that it follows that one manifold may have different dimensionality, depending on how we *order* it.

A manifold which has linear order and structure is called one-dimensional. A two-dimensional manifold is then a linearly ordered manifold of linearly ordered manifolds, .

Usually we speak about our 'space' of daily experiences as a three-dimensional manifold, but this is true only with reference to points, and not true with reference to lines or spheres. The manifold of all spheres in 'space' is, for instance, a four-dimensional manifold; so also is a manifold of lines.

Let us explain the line-dimensionality of our 'space' in terms of lines. A line can be given by two points—one, let us say, in the floor of our room, the other in the ceiling. Each of these points is given by two co-ordinates; it has two degrees of freedom; and so our 'space' is a four-dimensional (2×2) manifold *in lines.* This means that to distinguish any line in our 'space' from any other line we would have to have *four data.* Similarly, if we deal with spheres, a manifold made up of spheres requires four data, three for locating the centre and one giving the radius of the sphere. The above examples, of course, do not exhaust the structural possibilities.[1]

The term 'dimension' does not apply solely to what we call 'space'. The term applies to any manifold which we can order in some particular way. Manifolds or aggregates abound everywhere in our lives. The domain of colours, for instance, is a manifold; and so is the domain of tone, or of remembrances , . No manifold in itself has any dimensionality. To ascribe dimensionality to the manifold we must first order it and the number of its dimensionality, or its ascribed or discovered structure, may differ according to the principle of ordering used.

In discussing dimensionality we have two purposes. First, to dispel the semantic fright about this simple term; and, second, to suggest *means for visualization,* which for our purpose are of great neurological importance.

When we say that the world is structurally a four-dimensional manifold, we mean only that according to our experience and the structure of our nervous system, the world of our experience is represented by a fourfold order. We can order the events as to the right and to the left, forward and backward, up and down, and sooner and later. In our experience this fourfold order is completely united, and cannot be separated unless we deliberately choose to *neglect* some of these orderings.

Nor does it mean that all these dimensions are ‘identical’. We are accustomed, for instance, to consider the three dimensions of ‘space’ as ‘identical’, or at least equivalent. Is this true in life ? Can we disregard, for instance, the structural difference between vertical and horizontal ? If we did, quite probably, as Eddington remarks, we should come to an untimely end, and break our necks.

Obviously if we *visualize* our *plenum,* as made up of lines or particles, by necessity we visualize structurally a four-dimensional manifold. It should be noticed that a four-dimensional ‘absolute void’, or ‘absolute nothingness’, besides being non-sense, cannot be visualized at all, because it could have *no structure.*

We see that all metaphysical ‘fourth dimensions’ are not only non-sense, but usually indicate a pathological semantic disturbance. The intensity of such disturbances is often high, because it is entirely impossible for a sane person to deal with such meaningless noises. The victim is obsessed with attempts to do the impossible,—a semantically hopeless and painful task.

Such objectifications of terms are very dangerous and science should try, by proper emphasis, to eliminate them. Outside of science the term ‘dimension’ has no *meaning* and ought to be definitely abandoned in our speculations, for the sake of sanity.

The notion of ‘time’ as a ‘fourth dimension’ is by no means new. It appeared in a vague form centuries ago. The notion however was not formulated properly, and therefore was unworkable. Instead of helping science, it only hindered it.

Inspired by Einstein’s work, the mathematician Minkowski, whose work had been mainly in the theory of numbers, began to work at the theory of manifolds of any number of dimensions. In 1908 he delivered his famous and semantically epoch-making address on *Space and Time* which fused geometry and physics structurally. In this address he insisted that the connection between ‘space’ and ‘time’ as given by the Lorentz-Einstein formulae is not accidental but exhibits that inner connection or structure to which we had not paid enough attention.[2]

In our *experience,* ‘space’ and ‘time’ can never be entirely separated, as already explained, and so Minkowski combined them into a higher entity which is called the ‘Minkowski world’. In the world of experience the datum appears to be, not a place and a point of ‘time’, but the *event* or the *world point*—that is, a place at a definite *date.*

The graphic picture of a moving point is a *world-line.* Rectilinear uniform motion corresponds then to a straight world-line; accelerated motion, to one that is curved.

The *event* is the most elementary notion. We shall use it from now on in this work in the sense of a four-dimensional volume of space-time which is small in all four dimensions. We do not posit whether events themselves have structure or not, but it is preferable to assume that they have no space-time structure, which means that the event has no parts which are external to each other in space-time. The order of events is fourfold, as previously shown.

The aggregate or manifold of all point-events is then called the world. The point-events are given by four numbers representing the co-ordinates, three giving the 'space' co-ordinates, and the fourth the 'time' co-ordinate.

The term 'space-time continuum' or 'space-time manifold' is used often and implies that the numbers x, y, z, t, are to vary continuously.

In such a space-time continuum all happenings are structurally the intersections of world-lines, and if we could describe the world-lines of all points of the universe we would have a full account of the universe, 'past' and 'future'. We see that all physics, with the *rest of our problems,* must then be considered as a chapter of the *general* structural and semantic study of continuous manifolds of four dimensions.

But we are already acquainted with such theories. For instance, the internal theory of surfaces may be considered as a part of the subject in two and three dimensions. We have seen that different surfaces are characterized by the expression for the line element $ds^2=g_{11}dx_1^2+2g_{12}dx_1dx_2+g_{22}dx_2^2$, or by that group of transformations which leaves the line element invariant. We know already that in the [E] as well as riemannian geometries we have similar expressions and characteristic transformations.

If physics is to be considered a branch of the theory of four-dimensional manifolds, we should naturally look for some such transformations. The manifold represents the world, the *generalized*[*] theory of relativity gives the desired answer. Minkowski proposed a postulate, which he calls the *postulate of an absolute world,* or the world-postulate which asserts the *invariance of all the laws* of nature in relation to linear transformations, for which the function $x^2+y^2+z^2-c^2t^2$ is invariant

The reader is already familiar with the expression x^2+y^2, which gives the invariant length in [E] geometry in two dimensions, and $x^2+y^2+z^2$ which gives it in three dimensions. It would be natural to expect that in four dimensions we should have an expression of the type $x^2+y^2+z^2+t^2$ but in this case our expression is $x^2+y^2+z^2-c^2t^2$. It should be noticed that the above different types of expressions have different origins. The first two arise in pure geometry, and the last has its roots in physics. The problem was to bring an experimental expression into harmony with a familiar geometrical expression. Minkowski introduced the expression $ict=u_4$, where i is as usual the square root of minus one, ($i=\sqrt{-1}$). Then of course $-c^2t^2$ becomes $(ict)^2=u_4^2$.

[*] I use the term 'generalized' to embrace the unified field theory and eventually the quantum theory, although, for our purpose, I utilize only the special and general theory.

If we change the lettering, and denote $x=u_1$; $y=u_2$; $z=u_3$ then our expression $x^2+y^2+z^2-c^2t^2$ becomes $u_1^2+u_2^2+u_3^2+u_4^2$, a simple formula for distance in four-dimensional geometry. It would not be profitable for us to speculate upon this substitution; it was introduced merely for the sake of mathematical, verbal treatment and can be easily translated back into the usual terms of c and t.

We have already seen that the expression $x^2+y^2+z^2-c^2t^2=x'^2+y'^2+z'^2-c^2t'^2$ is invariant under the Lorentz-Einstein transformation. This fact of invariance is fundamental, and it is well to convince ourselves that it is so. The Lorentz-Einstein transformation was $x'=\beta(x-vt)$, $y'=y$, $z'=z$, $t'=\beta(t-vx/c^2)$ where $\beta=1/\sqrt{(1-v^2/c^2)}$. As the co-ordinates y and z are equal in both systems, we can disregard them, and verify that $c^2t'^2-x'^2=c^2t^2-x^2$. Let us substitute for t' and x' the values given by the Lorentz-Einstein transformation. We have then:

$$c^2t'^2-x'^2=c^2\beta^2(t-vx/c^2)^2-\beta^2(x-vt)^2=c^2t^2\beta^2+c^2\beta^2v^2x^2/c^4-2c^2\beta^2tvx/c^2-\beta^2x^2-\beta^2v^2t^2+2\beta^2xvt=c^2t^2\beta^2(1-v^2/c^2)-x^2\beta^2(1-v^2/c^2)=c^2t^2(1-v^2/c^2)/(1-v^2/c^2)$$
$$-x^2(1-v^2/c^2)/(1-v^2/c^2)=c^2t^2-x^2;$$

since $\beta^2 = 1/(1-v^2/c^2)$. Similarly it is easy to show, if we take an event-particle, as, for instance, a momentary spark, which has the co-ordinates x_1, y_1, z_1, t_1, in one system of co-ordinates, say S, and let another event-particle occur in that system at x_2, y_2, z_2, t_2, that the formulae remain invariant. If we designate the distance between the two events by r, its value would be given by $r^2=(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2$

In a different system, S', moving uniformly relatively to S. r'^2 would in general not be equal to r^2, but the expression $r^2-c^2(t_2-t_1)^2$ would be equal to $r'^2-c^2(t'_2-t'_1)^2$,
$$r^2-c^2(t_2-t_1)^2=r'^2-c^2(t'_2-t'_1)^2$$

The above expression is called the *interval* and expresses a most fundamental structural characteristic; namely, that the interval is invariant for all systems in uniform relative motion. This result is quite general and independent of the relative orientation of the axes, or of the angle the velocity has to the axes. The interval plays in the theory of Einstein a similar role which the pythagorean rule played in the [E] geometry.

Because of the finite velocity of our measuring signals, our formulae must involve finite velocity. Therefore the interval is the only actual measurement which we can ever make in practice. Hence its fundamental semantic importance.

Eddington gives a very fine diagram in explaining how intervals are measured. I reproduce it herewith (Fig. 1).

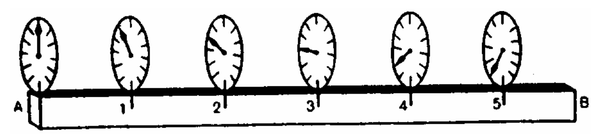

FIG. 1

The expression ct, where c is the velocity of light, 300,000 kilometres per second, gives us the *distance* travelled by light in the 'time' t. It is natural to regard the velocity of light, which is a constant and translates easily into the language of length, as a unit of 'time'. In the Minkowski world it is customary, because of its convenience, to regard 1 second as the equivalent of 300,000 kilometres and measure lengths or 'times' in seconds or kilometres indiscriminately.

Let us imagine a scale graduated in kilometres, and clocks whose faces are also graduated in kilometres (1/300,000 of a second). If the clocks are set correctly and we look at them from A the sum of the reading of any clock and the scale division beside it is one for all because the scale reading gives the correction for the 'time' taken by light, travelling with unit velocity, to reach A.

If we lay the scale in line with the two events and note the clock and scale readings, t_1 and x_1, of the first event, and the corresponding readings, t_2 and x_2, of the second event, then $s^2=(t_2-t_1)^2-(x_2-x_1)^2$ where s represents the 'interval' mentioned above.

If we set the scale moving in the direction AB then the divisions would have advanced to meet the second event and the difference (x_2-x_1) would be smaller. But this is *compensated,* because (t_2-t_1) also becomes altered. When A is advancing to meet the light coming from any of the clocks on the scale the light arrives too quickly, and the reading of the clock appears smaller.

The net result is, roughly, that it does not matter what uniform motion is given to the scale, the final results for the interval s are always equal.[3]

We can now understand the vital importance of the minus sign with the 'time' co-ordinate. In fact, if in our equations all the signs were plus, using the 'space' and 'time' of one observer, one value of s would be obtained; but using the 'space' and 'time' of another observer, a different value would be obtained. With the minus sign for the 'time' co-ordinate, we see that we can have values of s which are equal for all observers. If the distances increase, the 'time' element increases also, and so the *difference* may not be changed, but with the positive sign this would not be the case.

We see that the interval s represents something which concerns only the events under consideration. The corresponding entity in ordinary geometry is *distance,* which is independent of the accidental choice of co-ordinates. The minus sign makes the geometry of space-time *non-euclidean.*

To familiarize ourselves with what has been already explained about simultaneity and the geometry of space-time, we will work it out once more, but now by the Minkowski method.

It will be enough to use two dimensions, one represented on the X axis, the other on the T axis. Let us consider three points A, B, C, at rest in our system O on the X axis (Fig. 2). In our space-time they will be represented by three parallels to the T axis. Let C be midway between A and B so that $AC=CB$. Let us assume that light signals are sent in both directions from C at the moment $t=0$. We assume that the system is 'at rest', which means that the light signals propagate themselves to the right and to the left with equal velocities. Hence we can represent them by straight lines equally inclined to the *X* axis. These lines are called ‘light-lines’.

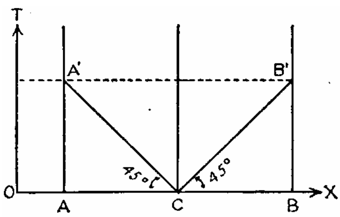

Fig. 2

The points *A'*, *B'* which are the intersection of the ‘world-lines’ of the points *A* and *B* with the light-lines give us the ‘times’ at which the signals arrive. It follows from the drawing that *A'B'* is parallel to the *X* axis, which means that *A'* and *B'* are ‘simultaneous’ (equal ‘times’). Let us now take another case in which our points *A, B, C,* move-uniformly with an equal velocity (Fig. 3). Their world-lines will also be parallel to each other but *inclined* to the axis. In the drawing the light-lines will be represented by similar lines but their intersections with the world-lines of *A* and *B* will not be on a parallel to the *X* axis, and so they will not be simultaneous.

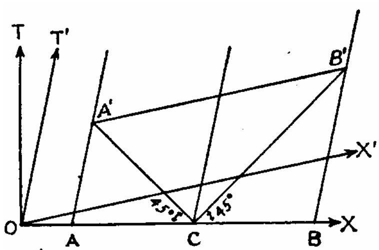

Fig. 3

We should notice that an observer who moved with the system in the direction *OX'* would be perfectly entitled to claim that *A'* and *B'* are *simultaneous to him.* His co-ordinate system would be in which the points *A'* and *B'* are on a parallel to *his X' axis* as he is *at rest* in his system *OX'T'*. The world-lines *A, B, C,* are parallel to the *T'* axis because the points are supposed to be at rest in this system and hence the *x'*s have equal values for all *t's.*

An important point should be noticed; namely, that we have only one space-time and that the indefinitely numerous ways different observers partition their ‘space’ and ‘time’ represent merely the indefinitely many ways in which it can be partitioned. If we keep the whole of it under consideration we see that we cannot divide it into ‘space’ *and* ‘time’, as any subdivision has both aspects.[4]

The Minkowski method of representation makes the change in our measurements of length, as given by the Lorentz-Einstein transformation, very obvious.

A measuring rod is not purely a ‘spatial’ configuration, as in the actual world such a thing does not exist, but it is a space-time configuration.

Every point of the rod exists at each moment of ‘time’. We see that in space-time we cannot represent our rod as a segment on the *X* axis but must represent it structurally as a *strip* in the *XT* plane. We assume here for simplicity that the rod is one-dimensional (Fig. 4).

A rod which is at rest in a system is represented by a strip parallel to the *T* axis. If it is moving, its strip is inclined to the *T* axis. The ‘contraction’ does not affect the strip at all but it is rather a section cut out of the *X* axis. In actual experience, it is only the strip as a manifold of world points which has *physical reality*, and not the cross sections, which, as we see, are not equal on different axes. The 'contraction' is not a change in 'physical reality' but merely a consequence of our way of regarding things. We see that the notorious argument as to whether the 'contraction' is 'real' or 'apparent' is based on a misunderstanding. Born gives an excellent example. If we slice a cucumber in different directions it is fallacious to argue that the smallest slice which is perpendicular to the axis is the 'real' one and the larger oblique slices only 'apparent'. Similarly in the Einstein theory, a rod has various lengths according to the motion of the observer. One of these lengths, the static length, is the greatest, but it is no more 'real' than any other. Similar remarks can be made about 'time'.

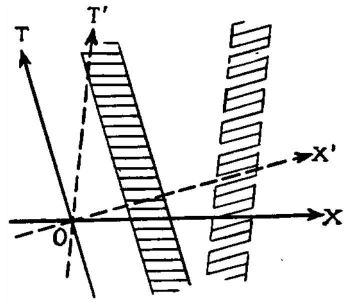

FIG. 4

Attention should be given to one extremely important semantic point concerning the Minkowski four-dimensional world. We already know that for our nervous systems the passing from dynamic to static, and vice versa, is a most vital structural problem. The first step of this translation has already been given in the notion of the 'variable'. The calculus carried it a step further. In the Minkowski world we reach the complete solution of the problem.

As Keyser points out in his *Mathematical Philosophy*, we had two verbal methods of dealing with 'time'. One was the method of Newton, the method of the structural importation of 'time'. From the objective dynamic world of the lower order abstractions, 'time' is imported into the static world of the higher order abstractions. We import it with 'motion', we say things 'move'. Such language is structurally unsatisfactory, even on the earlier level of our development. It hampers analysis, and is contrary to the structure and function of the *human* nervous system. It breeds tremendous metaphysical impasses, and is ultimately based on semantic disturbances due to identification.

If we introduce dynamic, shifting entities into static higher order abstractions, rationality is impossible and we drift toward mysticism.

A very real semantic problem appears here. We want to give the best possible account of the structurally *dynamic* world around us; yet our higher order abstractions are structurally *static*, and for their proper working they must use static means. Here seemed to be an impasse which for milleniums had defied solution. 'Philosophers' of different schools were preaching and teaching that we should never be able to be 'rational' and understand this world and ourselves. Anti-intellectual schools began to flourish, to the bewilderment of all.

The issue, after all, was simple, the moment some one discovered and stated it. We did not need to change either the world around us or ourselves;

we simply had to discover a structurally new method of dealing with the old problems without changing them.

The new method is given by Einstein and Minkowski. Instead of making the static world of higher order abstractions dynamic, which cannot be done at all without producing semantic disturbances, they invented structural methods for dealing with the dynamic world by static means. The key was found in the handling of the troublesome factor, ‘time’. Minkowski decided to put ‘time’ in its proper place by introducing the structurally new four-dimensional world.

In the case of particle P, we habitually used to say that the point P at an instant t was at a ‘space’ point (x, y, z). At the instant t' it was at the point (x', y', z'), . We always needed four numbers, which gave us the where and when in respect to some frame of reference.

As we have seen, Minkowski decided to look structurally at this tetrad of four numbers (x, y, z, t) ‘as-a-whole’. In other words, he placed himself on a higher level of abstraction. He took under consideration the older results, combined them, and called the combination by one single name, the ‘world point’. Such a world point has also four numbers (not 3 plus 1, but just 4). A world made up of such points is a four-dimensional world in which all the points co-exist. The flux of the lower order abstractions and ‘time’ is abolished. There is no more ‘motion’ in a ‘flow of time’. In such a world the term ‘where’ has completer structural meanings, it has absorbed the *when.* If we ask, where in such a world the particle P is, we answer, at the point (x, y, z, t). Where is the particle P' ? At the point (x', y', z', t').

We see that the particles of such a world *are never ‘the same’;* they do not ‘change’ or ‘pass’; they co-exist, and all is *static.* In this way the three-dimensional dynamics become four-dimensional statics.

It should be noticed that we are now dealing with a language of new structure, uniquely befitting the structure and function of our nervous system. Of course we have altered nothing in the world around us. The example of the moving picture and the static film as given before is an excellent example of this structural innovation.

The fact that in this new world nothing repeats itself because it has a different date, unless the time-lines are closed, has very far-reaching consequences, of which we have already spoken and which we will analyse in more detail later on.