CHAPTER XIX

MATHEMATICS AS A LANGUAGE OF A STRUCTURE SIMILAR TO THE STRUCTURE OF THE HUMAN NERVOUS SYSTEM

In recent times the view becomes more and more prevalent that many branches of mathematics are nothing but the theory of invariants of special groups. S. LIE

A natural law,—if, strictly speaking, there be such a thing outside the conception thereof,—is fundamentally nothing more nor less than a constant connection among inconstant phenomena: it is, in other words, an invariant relation among variant terms. (264) CASSIUS J. KEYSER

Whatsoever things are invariant under all and only the transformations of some group constitute the peculiar subject-matter of some (actual or potential) branch of knowledge. (264) CASSIUS J. KEYSER

The general laws of nature are to be expressed by equations which hold good for all systems of co-ordinates, that is, are co-variant with respect to any substitutions whatever (generally co-variant). (155) A. EINSTEIN

The things hereafter called tensors are further characterized by the fact that the equations of transformation for their components are linear and homogeneous. Accordingly, all the components in the new system vanish if they all vanish in the original system. If, therefore, a law of nature is expressed by equating all the components of a tensor to zero, it is generally covariant. By examining the laws of the formation of tensors, we acquire the means of formulating generally covariant laws. (155) A. EINSTEIN

The thalamus is a centre of affective reactivity to sensory stimuli, while the cortex is an apparatus for discrimination. (411) HENRI PIÉRON

Section A. Introductory.

It becomes increasingly evident that we have come to a linguistic impasse, reflected in our historical, cultural, economic, social, doctrinal. , impasses, all these issues being interconnected. The structural linguistic aspect is the most fundamental of them all, as it underlies the others and involves the *s.r*, or psycho-logical responses to words and other events in connection with meanings.

One of the benefits of building a system on undeniable negative premises is that many older and controversial problems become relatively simple and often uncontroversial, disclosing an important psycho-logical mechanism. Such formulations have often the appearance of the ‘discovery of the obvious’; but it is known, in some quarters, that the discovery of the obvious is sometimes useful, not always easy, and often much delayed; as, for instance, the discovery of the equality of gravitational and inertial mass, which has lately revolutionized physics.

As words are *not* the things we are talking about, the only possible link between the objective world and the verbal world is structural. If the two structures are similar, then the empirical world becomes intelligible to us—we ‘understand’, we can adjust ourselves, . If we carry out verbal experiments and predict, these predictions are verified empirically. If the two structures are not similar, then our predictions are not verified —we do not ‘know’, we do not ‘understand’, the given problems are ‘unintelligible’ to us. , we do not know what to do to adjust ourselves, .

Psycho-logically, in the first case we feel security, we are satisfied, hopeful. ; in the second case, we feel insecure, a floating anxiety, fear, worry, disappointment, depression, hopelessness, and other harmful *s.r* appear. The considerations of structure thus disclose an unexpected and powerful semantic mechanism of individual and collective happiness, adjustment. , but also of tragedies, supplying us with *physiological* means for a certain amount of desirable control, because relations and structure represent fundamental factors of all meanings and evaluations, and, therefore, of all *s.r.*

The present increasing world unrest is an excellent example of this. The structure of our old languages has shaped our *s.r* and suggested our doctrines, creeds. , which build our institutions, customs, habits, and, finally, lead fatalistically to catastrophes like the World War. We have learned long ago, by repeated sad experience, that predictions concerning human affairs are not verified empirically. Our doctrines, institutions, and other disciplines are unable somehow to deal with this semantic situation, and hence the prevailing depression and pessimism.

We hear everywhere complaints of the stupidity or dishonesty of our rulers, as already defined, without realizing that although our rulers are admittedly very ignorant, and often dishonest, yet the most informed, gifted, and honest among them cannot predict or foresee happenings, if their arguments are performed in a language of a structure dissimilar to the world *and* to our nervous system. Under such conditions, calling names, even under provocations, is not constructive or helpful enough. Arguments in the languages of the old structure have led fatalistically to systems which are structurally ‘un-natural’ and so must collapse and impose unnecessary and artificial stress on our nervous system. The self-imposed conditions of life become more and more unbearable, resulting in the increase of ‘mental’ illness, prostitution, criminality, brutality, violence, suicides, and similar signs of maladjustment. It should never be forgotten that human endurance has limits. Human ‘knowledge’ shapes the human world, alters conditions, and other features of the

environment—a factor which does not exist to any such extent in the animal world.

We often speak about the influence of heredity, but much less do we analyse what influence environment, and particularly the *verbal environment,* has upon us. Not only are all doctrines verbal, but the structure of an old language reflects the structural metaphysics of bygone generations, which affect the *s.r.* The vicious circle is complete. Primitive mythology shaped the structure of language. In it we have discussed and argued our institutions, systems. , and so again the primitive structural assumptions or mythologies influenced them. It should not be forgotten that the affective interplay, interaction, interchange is ever present in human life, excepting, perhaps, in severe and comparatively rare (not in all countries) 'mental' ills. We can stop talking, we can stop reading or writing, and stop any 'intellectual', interplay and interaction between individuals, but we cannot stop or entirely abolish some *s.r.*

A structural linguistic readjustment will, it is true, result in making the majority of our old doctrines untenable, leading also to a fundamental scientific revision of new doctrines and systems, affecting all of them and our *s.r* in a constructive way. It is incorrect, for instance, to use the terms 'capitalism' as opposed to 'socialism', as these terms apply to different non-directly comparable aspects of the human problem. If we wish to use a term emphasizing the *symbolic character* of human relations, we can use the term 'capitalism', and then we can contrast directly individual, group, national, international. , capitalisms. If we want to emphasize the psycho-logical aspects, we can speak of individualism versus socialism, . Obviously, in life the issues overlap, but the verbal implications remain, preventing clarity and inducing inappropriate *s.r* in any discussion.

In vernacular terms, there is at present a 'struggle' and 'competition' between two entirely different 'industrialisms' and two different 'commercialisms', based ultimately on two different forms of 'capitalism'. One is the 'individual capitalism', rapidly being transformed into 'group capitalism', in the main advanced theoretically to its limits in the United States of America and to a lesser extent in the rest of the civilized world, and 'social capitalism', proclaimed in the United Socialistic Soviet Republics. Both these extreme tendencies, connected also with semantic disturbances, are due to a verbal or doctrinal 'declaration of independence' of two, until lately, much isolated countries. The United States of America proclaimed the doctrine that man is 'free and independent', while, in fact, he is *not* free, but is inherently *interdependent.* The Soviets accepted uncritically an unrevised antiquated doctrine of the

‘dictatorship of the proletarians’. In *practice*, this would mean the dictatorship of unenlightened masses, which, if left *actually to their creeds*, and deprived of the *brain-work* of scientists and leaders, would revert to primitive forms of animal life. Obviously, these two extreme creeds violate every typically *human* characteristic. We are interdependent, time-binders, and we are interdependent because we possess the higher nervous centres, which complexity animals do not possess. Without these higher centres, we could not be human at all; both countries seem to disregard this fact, as in both the brain-work is exploited, yet the brain-workers are not properly evaluated. The ignorant mob, with its historically and psycho-logically cultivated animalistic *s.r*, retards human progress and agreement. Leaders do not lead, but the majority play down to the mob psycho-logics, in fear of their heads or stomachs.

In both countries, the *s.r* are such that brain-work, although commercially exploited, is not properly evaluated, and is still persecuted here and there. For instance, in the United States of America, we witness court trials and resolutions against the work of Darwin, in spite of the fact that without some theory of evolution most of the natural sciences, medicine included, would be impossible. In Russia, we find decrees against researches in pure science, without which *modern* science is impossible. Both countries seemingly forget that all ‘material’ progress among humans is due uniquely to the *brain-work* of a few mostly underpaid and overworked workers, who exercise properly their higher nervous centres. With science getting hold of problems of *s.r* and sanity, our human relations and individual happiness will also become the subject matter of scientific enquiry. If international and *inter*dependent brain-workers produce discoveries and inventions, any one, even of the lowest development, can use or misuse their achievements, no matter what ‘plan’, or ‘no-plan’, is adopted. Both countries seem at present not to understand that a great development of mechanical means and the application of scientific achievements exclusively for animal comfort fail to lead to greater happiness or higher culture, and that, perhaps, indeed, they lead in just the opposite direction. Personally, I have no doubt that some day they will understand it; but an earlier understanding of this simple semantic fact would have saved, in the meantime, a great deal of suffering, bewilderment, and other semantic difficulties to a great number of people, if the rulers in both countries would be enlightened enough and could have foreseen it soon enough.

The future will witness a struggle between the individual and group capitalism, as exemplified in the United States of America, and the collective or social capitalism, as exemplified in the Soviet Republics. It does

not require prophetic vision to foresee that some trends of history are foregone conclusions because of the structure of the human nervous system. As trusts or groups have replaced the theoretically ‘individual’ capitalism in the United States of America, so will the state capitalism replace the trusts, to be replaced in its turn by international capitalism.

We are not shocked by the international character of science. We are not ‘100 per cent patriotic’ when it comes to the use in daily life of discoveries and inventions of other nations. Science is a semantic product of a *general human symbolic characteristic;* so, naturally, it must be general and, therefore, ‘international’. But ‘capitalism’ is also a unique and *general* semantic product of symbolism; it is also a unique product of the human nervous system, dependent on mathematics, and, as such, by its inherent character, must become some day international. There is no reason why our *s.r* should be disturbed in one case more than in the other. The ultimate problem is not whether to ‘abolish capitalism’ or not, which will never happen in a symbolic class of life, but to transfer the control from private, socially irresponsible, uncontrolled, and mostly ignorant, leaders to more responsible, *professionally trained,* and socially controlled *public servants,* not bosses. If a country cannot produce honest, intelligent, and scientifically trained public men and leaders, that is, of course, very disastrous for its citizens; but this is not to be proclaimed as a rule, because it is an exception. Thus, in the Soviet Republics, graft is practically non-existent in the sense that it exists in the United States; but the mentality of the public men is practically at a similar standstill because of a deliberate minimizing of the value of brain-work. I wonder if it is realized at all, in either country, that *any* ‘manual worker’, no matter how lowly, is hired *exclusively* for his *human brain,* his *s.r,* and *not* primarily for his hands !

The only problem which the rest of mankind has to face is how this struggle will be managed and how long it will last, the outcome admitting of no doubt, as the ruthless elimination of individual capitalism by group capitalism (trusts) in the United States is an excellent example. In the Soviet Republics, they simply have gone further, but in a similar direction. Struggles mean suffering; and we should reconcile ourselves to that fact. If we want the minimum of suffering, we should stop the animalistic methods of contest. Human methods of solving problems depend on higher order abstractions, scientific investigations of structure and language, revision of our doctrines. , resulting in peaceful adjustment of living facts, which are actualities whether we like it or not. If we want the maximum of suffering, let us proceed in the stupid, blind, animalistic and unscientific way of trial and error, as we are doing at present.

My aim is not to be a prophet, but to analyse different structural and linguistic semantic issues underlying all human activities, and so to produce material which may help mankind to *select* their lot *consciously*. What they will do is not my official concern, but it seems that both countries, which have so much in common, and which are bound to play an important role in the future of mankind, owing to their numbers, their areas, and their natural resources. , will have to pay more attention to the so-called 'intellectual' issues, or, more simply, not disregard the difference between the reactions of infants and adults. Otherwise, very serious and disastrous cultural results for all of us will follow.

The problems of the world 1933 are acute and immediate, overloaded with confusion, bitterness, hopelessness, and other forms of semantic disturbances. Without some means—and, in this case, scientific and physiological means—to regulate our *s.r*, we shall not be able to solve our problems soon enough to avoid disasters. The similarity in structure of mathematics, and our nervous system, once pointed out and *applied*, gives us a unique means to regulate the *s.r*, without which it is practically impossible to analyse dispassionately and wisely the most pressing problems of immediate importance.

The present investigation shows that the old languages which, in structure, are *not* similar to the world and our nervous system, have automatically reflected their structure on our doctrines, creeds, and habits, *s.r*, and also on those man-made institutions which result from verbal arguments. These, in turn, shape further *s.r* and, as long as they last, control our destinies.

Four important issues could be shown in detail, but, for lack of space, I give only a suggestive sketch of them here.

1) In the *A*-system, all our existing older sub-systems, with all their benefits as well as shortcomings, follow as an *A* psycho-logical structural semantic necessity.

2) The tremendous handicap for any new and less deficient systems consists in the fact that such systems lack new constructive ∞-valued semantics, and are carried on the one side by linguistic two-valued arguments in the language of old *el* structure; yet they aspire 'emotionally' to something new and better, while the two cannot be reconciled.

3) An argument carried on in the old *el* and two-valued way, no matter how fundamentally true and eventually beneficial, can be easily defeated on verbal grounds if it follows the old structure of language. Our decisions are never well-grounded psycho-logically, and so can never command the respect or achieve the reliability of scientific reason-

ing. That is why we are groping—the only method possible under such conditions being the animalistic trial-and-error methods, swaying masses by inflammatory speeches because reason has nothing to offer, being tied up by the old verbal structure to the older consequences based on animalistic and fundamentally false-for-man assumptions.

4) In the old *A, el,* two-valued system, agreement is theoretically impossible; so one of the main, and perhaps revolutionary, semantic departures from the old system is the fact that a *non-el* ∞-valued $\overline{A}$-system, based on fundamental *negative* premises, leads to a theory of *universal agreement,* which is based on a structural revision of our languages, producing new and undisturbed *s.r,* which eliminate the copying of animals in our nervous reactions.

The subject matter of this chapter divides, naturally, into three interconnected semantic parts. In the first, we shall recall a few general notions, known in the main but seldom taken into consideration, reformulated in a language of different structure. In the second, I shall indicate how the most important mathematical disciplines, which traditionally and, in the opinion of the majority, could hardly be called mathematical, represent a scientific and exact formulation of the *general* 'thinking' process. In this connection, a few words will be said about the theory of aggregates, and a little more about the theory of groups. This latter theory, with its implications and applications, leads to a reformulation of mathematics on quite obvious psycho-*logical* grounds, bringing mathematics into the closest relationship to the general processes of mentation. Finally, in the third part, I shall indicate the astonishing and quite unexpected *physiological* fact of the similarity of the structure of mathematics with the structure and function of our nervous system.

The intelligent layman should be reminded that, although he needs to know *about* mathematics, the minimum given here, supplemented, perhaps, by a few most elementary and fascinating books on mathematical philosophy, given in the bibliography to this volume, yet he does not need, and probably never will need, more technical mathematics than is given in the high schools and supplemented by the fundamental notions of the differential calculus. For directly we treat all languages, mathematics included, from a more general (and, at present, perhaps, the most general) aspect; namely, *structure;* the reader will obtain all the essential psycho-logical benefits of modern science by absorbing the $\overline{A}$-system and habits, which will result in completely novel standards of evaluation and distinctly modern and adult *s.r*.

The last is of extreme and unrealized importance. In fact, its importance cannot be fully appreciated until we actually acquire such reactions,

because only then shall we have semantic disturbances eliminated, so that all problems can be analysed properly, and, therefore, agreement must be reached.

The future generations, of course, will have no difficulties whatsoever in establishing the healthy *s.r*; neither at present have very young children. These do not need such treatises as the present work. But, before the grown-up parents or teachers can train their children, they must first unlearn a great deal and train themselves to new habits involving the $\overline{A}$ *standards* of evaluation. So, for them, such a book, in order to be convincing, must deal with the foundations of their difficulties. The last task is difficult for the writer as well for the reader.

What has been said here does not apply, I am sorry to say, to *professional* 'philosophers', 'logicians', 'psychologists', psychiatrists, and teachers. These, to be adequate at all for their responsible and difficult professional duties, *must become thoroughly acquainted* with *structure* in general, and with the *structure of mathematics* in particular, as factors in *s.r*, and must work out the present outline much further.

Section B. General.

Mathematics in the twentieth century is characterized by an enormous productiveness, by the revision of its foundations, and the quest for rigour, all of which implies material of great and unexplored psycho-logical value, a result of the activity of the human nervous system. Branches of mathematics, as, for instance, mathematical 'logic', or the analytical theory of numbers, have been created in this period; others, like the theory of function, have been revised and reshaped. The theory of Einstein and the newer quantum mechanics have also suggested further needs and developments.

Any branch of mathematics consists of propositional functions which state certain structural relations. The mathematician tries to discover new characteristics and to reduce the known characteristics to a dependence on the smallest possible set of constantly revised and simplest structural assumptions. Of late, we have found that no assumption is ever 'self-evident' or ultimate.

To those structural assumptions, we give at present the more polite name of postulates. These involve undefined terms, not always stated explicitly, but always present implicitly. A *postulate system gives us the structure of the linguistic scheme.* The older mathematicians were less particular in their methods. Their primitive propositional functions or postulates were less well investigated. They did not start explicitly with undefined terms. The twentieth century has witnessed in this field

a marked progress in mathematics, though much less in other verbal enterprises; which accounts for the long neglect of the structure of languages. Without tracing down a linguistic scheme to a postulate system, it is extremely difficult or impossible to find its structural assumptions.

A peculiarity of modern mathematics is the insistence upon the formal character of all mathematical reasoning, which, with the new *non-el* theory of meanings, ultimately should apply to all linguistic procedures.

The problems of 'formalism' are of serious and neglected psycho-logical importance, and are connected with great semantic dangers in daily life if associated with the lack of consciousness of abstracting; or, in other words, when we confuse the orders of abstractions. Indeed, the majority of 'mentally' ill are *too formal* in their psycho-logical, one-, two-, or few-valued processes and so cannot adjust themselves to the ∞-valued experiences of life. Formalism is only useful in the search for, and test of, structure; but, in that case, the consciousness of abstracting makes the attitude behind formal reasoning ∞*-valued and probable*, so that semantic disturbances and shocks in life are avoided. Let us be simple about it: the mechanism of the semantic disturbance, called 'identification', or 'the confusion of orders of abstractions' in general, and 'objectification' in particular, is, to a large extent, dependent on two-valued formalism without the consciousness of abstracting.

In mathematics, formalism is uniquely useful and necessary. In mathematics, the formal point of view is pressed so far as to disclaim that any meanings, in the ordinary sense, have been ascribed to the undefined terms, the emphasis being on the postulated relations between the undefined terms. The last makes the majority of mathematicians able to adjust themselves, and mathematics extremely general, as it allows use to ascribe to the mathematical postulates an indefinite number of meanings which satisfy the postulates.

This fact is not a defect of mathematics; quite the opposite. It is the basis of its tremendous practical value. It makes mathematics a linguistic scheme which embodies the possibility of perfection, and which, no doubt, satisfies semantically, at each epoch, the great majority of properly informed individual Smiths and Browns. There is nothing absolute about it, as all mathematics is ultimately a product of the human nervous system, the best product produced at each stage of our development. The fact that mathematics establishes such linguistic relational patterns without specific content, accounts for the generality of mathematics in applications.

If mathematics had physical content or a definite meaning ascribed to its undefined terms, such mathematics could be applied only in the given case and not otherwise. If, instead of making the mathematical statement that one and one make two, without mentioning what the one or the two stands for, we should establish that one apple and one apple make two apples, this statement would not be applied safely to anything else but apples. The generality would be lost, the validity of the statement endangered, and we should be deprived of the greatest value of mathematics. Such a statement concerning apples is not a mathematical statement, but belongs to what is called 'applied mathematics', which has content. Such experimental facts as that one gallon of water added to one gallon of alcohol gives *less* than two gallons of the mixture, do not invalidate the mathematical statement that one and one make two, which remains valid by definition. The last mentioned experiment with the 'addition' of water to alcohol is a deep sub-microscopic structural characteristic of the empirical world, which must be discovered at present by experiment. The most we can say is that we find the above mathematical statement applicable in some instances, and non-applicable in others.

Not assigning definite meanings to the undefined terms, mathematical postulates have variable meanings and so consist of propositional functions. Mathematics must be viewed as a manifold of patterns of exact relational languages, representing, at each stage, samples of the best working of the human 'mind'. The application to practical problems depends on the ingenuity of those desiring to use such languages.

Because of these characteristics, mathematics, when studied as a form of human behaviour, gives us a wealth of psycho-logical and semantic data, usually entirely neglected.

As postulates consist of propositional functions with undefined terms, all mathematical proof is formal and depends exclusively on the form of the premises and not on special meanings which we may assign to our undefined terms. This applies to all 'proof'. 'Theories' represent linguistic structures, and must be proved on semantic grounds and never by empirical 'facts'. Experimental facts only make a theory more plausible, but no number of experiments can 'prove' a theory. A proof belongs to the *verbal* level, the experimental facts do not; they belong to a different order of abstractions, not to be reached by language, the connecting link being *structure,* which, in languages, is given by the systems of postulates.

Theories or doctrines are always linguistic. They formulate something which is going on inside our skin in relation to what is going on

on the un-speakable levels, and which is not a theory. Theories are the rational means for a rational being to be as rational as he possibly can. As a fact of experience, the working of the human nervous system is such that we have theories. Such was the survival trend; and we must not only reconcile ourselves with this fact, but must also investigate the structure of theories.

Theories are the result of extremely complex cyclic chains of nerve currents of the human nervous system. Any semantic disturbance, be it a confusion of orders of abstractions, or identification, or any of their progeny, called ‘elementalism’, ‘absolutism’, ‘dogmatism’, ‘finalism’. , introduces some deviations or resistances, or semantic blockages of the normal survival cycles, and the organism is at once on the abnormal non-adjustment path.

The structure of protoplasm of the simplest kind, or of the most elaborate nervous system, is such that it abstracts and reacts in its own specific way to different external and internal stimuli.

Our ‘experience’ is based normally on abstractions and integrations of different stimuli by different receptors, with different and specific reactions. The eye produces its share, and we may see a stone; but the eye does not convey to us the *feel* of weight of the stone, or its temperature, or its hardness, . To get this new wisdom, we need other receptors of an entirely different kind from those the eye can supply. If the eye plays some role in establishing the weight, for instance, without ever giving the *actual feel* of weight, it is usually misleading. If we would try to lift a pound of lead and a pound of feathers, which the balance would register as of equal weight, the pound of lead would feel heavier to us than the pound of feathers. The eye saw that the pound of lead is smaller in bulk, and so the doctrinal, semantic, and muscular expectation was for a smaller weight, and so, by contrast, the pound of lead would appear unexpectedly heavy.

As the eye is one of the most subtle organs, in fact, a part of the brain, science is devising methods to bring all other characteristics of the external world to direct or indirect inspection of the eye. We build balances, thermometers, microscopes, telescopes, and other instruments, but the character and *feel* of weight, or warmth. , must be supplied directly by the special receptors, which uniquely can produce the special ‘sensations’. The swinging of the balance, or the rise of the column of the thermometer, establishes most important *relations,* but does not give the immediate specific and un-speakable feel of ‘weight’ or of ‘warmth’. Our first and most primitive contact with a stone, its feel. , is a personal abstraction from the object, full of characteristics supplied by the

peculiarities of the special receptors. Our primitive picture ‘stone’ is a summary, an integration, of all these separate ‘sense’ abstractions. It is an abstraction from many abstractions, or an abstraction of a higher order.

Theories are relational or structural verbal schemes, built by a process of high abstractions from many lower abstractions, which are produced not only by ourselves but by others (time-binding). Theories, therefore, represent the shortest, simplest structural summaries and generalizations, or the highest abstractions from individual experience and through symbolism of racial past experiences. Theories are mostly not an individual, but a collective, product. They follow a more subtle but inevitable semantic survival trend, like all life. Human races and epochs which have not revised or advanced their theories have either perished, or are perishing.

The process of abstracting in different orders being inherent in the human nervous system, it can neither be stopped nor abolished; but it can be deviated, vitiated, and forced into harmful channels contrary to the survival trend, particularly in connection with pathological *s.r*. No one of us, even when profoundly ‘mentally’ ill, is free from theories. The only selection we can make is between antiquated, often primitive-made, theories, and modern theories, which always involve important semantic factors.

The understanding of the above is of serious importance, as, by proper selection of theories, all wasteful semantic disturbances, which lead even to crimes, and such historical examples of human un-sanity as the ‘holy inquisition’, burning at the stake, religious wars, persecution of science, the Tennessee trial. , could have been avoided.

Whenever *any one* says *anything*, he is indulging in theories. A similar statement is true of writing or ‘thinking’. We *must* use terms, and the very selection of our terms and the structure of the language selected reflect their structure on the subject under discussion. Besides, words are not the events. Even simple ‘descriptions’, since they involve terms, and ultimately undefined terms, involve structural assumptions, postulates, and theories, conscious or unconscious—at present, mostly the latter.

It is very harmful to sanity to teach a disregard for theories or doctrines and theoretical work, as we can never get away from them as long as we are humans. If we disregard them, we only build for ourselves semantic disturbances. The difference between morbid and not so obviously morbid confusions of orders of abstractions is not very clear. The strong affective components of such semantic disturbances must

lead to absolutism, dogmatism, finalism, and similar states, which are semantic factors out of which states of un-sanity are built.

We know that we must start with undefined terms, which may be defined at some other date in other undefined terms. At a given date, our undefined terms must be treated as postulates. If we prefer, we may call them structural assumptions or hypotheses. From a theoretical point of view, these undefined terms represent not only postulates but also variables, and so generate propositional functions and not propositions. In mathematics, these issues are clear and simple. Every theory is ultimately based on postulates which consist of propositional functions containing variables, and which express relations, indicating the structure of the scheme.

It appears that the main importance of the linguistic higher order abstractions is in their *public* character, for they are capable of being transmitted in neural and extra-neural forms. But our private lives are influenced also very much by the lower order abstractions, 'feelings', 'intuitions',. These can be, should be, but seldom are, properly influenced by the higher order abstractions. These 'feelings'. , are personal, un-speakable, and so are non-transmittable. For instance, we cannot transmit the actual feeling of pain when we burn ourselves; but we can transmit the invariant relation of the extremely complex fire-flesh-nerve-pain manifold. A relation is present empirically, but also can be expressed by words. It seems important to have means to translate these higher order abstractions into lower, and this will be the subject of Part VII.

Section C. The psycho-logical importance of the theory of aggregates and the theory of groups.

Starting with the $\overline{A}$ denial of identity, we were compelled to consider structure as the only possible link between the empirical and the verbal worlds. The analysis of structure involved relations and *m.o* and multi-dimensional order, and, ultimately, has led us to a semantic definition of mathematics and numbers. These definitions make it obvious that all mathematics expresses general processes of mentation *par excellence.* We could thus review all mathematics from this psycho-logical point of view, but this would not be profitable for our purpose; so we will limit ourselves to a brief sketch connected with the theory of aggregates and the theory of groups, because these two fundamental and most general theories formulate in a crisp form the general psycho-logical process, and also show the mechanism by which all languages (not only mathematics) have been built. Besides, with the exception of a few specialists, the general public is not even aware of the existence of such

disciplines which depart widely from traditional notions about mathematics. They represent most successful and powerful attempts at building exact relational languages in subjects which are on the border-line between psycho-logics and the traditional mathematics. Because they are exact, they have been embodied in mathematics, although they belong just as well to a general science of relations, or general semantics, or 'psychology', or 'logic', or scientific linguistics and psychophysiology. There are other mathematical disciplines, as, for instance, analysis situs, or the 'algebra of logic'. , to which the above statements apply; but, for our present purposes, we shall limit ourselves to the former two.

Dealing with the theory of aggregates, I will give only a few definitions taken from the *Encyclopaedia Britannica*, with the purpose of drawing the attention of the 'psychologists', and others, to those psycho-logical data.

The theory of aggregates underlies the theory of function. An aggregate, or manifold, or set, is a system such that: (1) It includes all entities to which a certain characteristic belongs; and (2) no entity without this characteristic belongs to the system; (3) any entity of the system is permanently recognizable as distinct from other entities.

The separate entities which belong to such a collection, system, aggregate, manifold, or set are called elements. We assume the possibility of selecting at pleasure, by a definite process or law, one or more elements of any aggregate *A*, which would form another aggregate *B*, .

The above few lines express how the human 'thought' processes work and how languages were built up. It is true that the exactness imposes limitations, and so the mathematical theories are not expressed in the usual antiquated 'psychological' terms, although they describe one of the most important psycho-logical processes.

Lately, the theory of aggregates has led to a weighty question: Does one of the fundamental laws of old 'logic'; namely, the two-valued law of the 'excluded third' (A is either B or not B), apply in all instances ? Or is it valid in some instances and invalid in others ?

This problem is the psycho-logical kernel of the new revision of the foundation of mathematics, which has lately been considerably advanced by Professor Lukasiewicz and Tarski with their many-valued 'logic', which merges ultimately with the mathematical theory of probability; and on different grounds has perhaps been solved in the present *non-el,* $\overline{A}$-system.

The notion of a group is psycho-logically still more important. It is connected with the notions of transformation and invariance. Without giving formal definitions unnecessary for our purpose, we may say that

if we consider a set of elements *a, b, c.* , and we have a rule for combining them, say O, and if the result of combining any two members of the set is itself a member of the set, such aggregate is said to have the 'group property'.

Thus, if we take numbers or colours, for instance, and the rule which we accept is '+', we say that a number or a colour is transformed by this rule into a number or a colour, and so both possess the 'group property'. Obviously, by performing the given operation, we have transformed one element into another; yet some characteristics of our elements have remained invariant under transformation. Thus, if 1 is a number and 2 is a number, the operation '+' transforms 1 into 3, since 1 + 2 = 3; but 3 has the character of being a number; so this characteristic is preserved or remains invariant. Similarly, with colours, if we add colours, these are transformed, but remain colours, and so both sets have the 'group property'. Keyser suggests that the 'mental' processes have the group property, which is undoubtedly true.[1]

The role this theory plays in our language is of great importance, because in it we find a method of search for structure, and a method by which we can establish a similarity of structure between the un-speakable objective level and the verbal level, based on invariance of relations which are found or discovered in both.

The role of groups in physical theory is best described by quoting Professor Rainich. (Remarks in brackets are mine.) 'A physicist, we may take it, is a person who measures according to certain rules. Let us denote by *a* the number he obtained in a given situation by applying the rule number one, by *b* the number obtained in the same situation by measuring according to rule number two and so on *(a* may be e.g. the volume, *b* the pressure, c the temperature of gas in a given container). The physicist finds further that the results of measurements of the same kind undertaken in different situations satisfy certain relations, we may write, for instance:

$$r(a,b)=c.$$

'A mathematician is busy deducing from some given propositions other propositions; this usually leads to numbers which we may call *A, B, C,* . . . . These numbers also satisfy certain relations, say

$$R(A,B)=C.$$

'Then comes, as Professor Weyl says, a messenger, a go-between who may be a mathematician or a physicist, or both, and says: "If you establish a correspondence between the physical quantities and the mathematical quantities, if you assign *A* to *a, B* to *b*, etc., the *same* relations

hold for the physical quantities as for the corresponding mathematical quantities so that $R \equiv r$." [*Similarity of structure.*]

'In the course of time new procedures of measurement are invented, some physical relations do not find their counterpart in the mathematical theory, the mathematical theory has to be patched up by introducing new quantities till too many quantities appear in it which do not correspond to physical quantities; then comes the phenomenological point of view and sweeps the theory out of applied mathematics—the theory becomes pure mathematics once more, and physicists begin to look around for a new theory. Everybody can find examples for this situation; it is enough to mention the Bohr atom which was not even mentioned today only fifteen years after its introduction.

'However the theory of groups which is being applied to physics is not just one of many mathematical theories of the character described above; its application is of a far more fundamental nature and we shall be able to indicate what it is by analysing further the scheme outlined above.

'It may happen, and in fact it happens often, that the same mathematical theory can be applied to the same physical facts in more than one way; for instance, instead of assigning to the physical quantities $a, b, \ldots$ the mathematical quantities $A, B, \ldots$ we might have assigned to them $A', B', \ldots$ with the same results, that is, the relations for physical quantities are the same as for the mathematical quantities corresponding to them now (think of space considered from the experimental point of view—and of coordinate geometry; different ways of establishing a correspondence result from different choices of coordinate axes). If this happens it means that the mathematical theory possesses a peculiar property, namely, that if A' is substituted for A, B' for B and so on, no relation of the type $R(A,B)=C$ which was correct before the substitution is destroyed; in other words, there are substitutions or transformations for which all relations are invariant. All such transformations constitute what we call a group; the existence and the properties of such a group present a very important characteristic of the mathematical theory. Moreover it is clear that if two different mathematical theories can be applied—in the sense described above—to the same physical theory, the groups of these two theories will be essentially the same, so that the groups reflect some of the most fundamental properties of physical systems.'[2]

The connection between groups and structure is described by Professor Shaw as follows: 'The first branch of dynamic mathematics is the theory of operations. It includes the general theory of operators

of any type and in particular the theory of groups of operators. The structure of such groups is evidently a study of form. It may often be exemplified in some concrete manner. Thus the groups of geometric crystals exemplify the structure of thirty-two groups of a discontinuous character, and the 230 space-groups of the composition of crystals exemplify the corresponding infinite discontinuous groups. The study of the composition series of groups, the subgroups and their relations, whether in the case of substitution groups, linear groups, geometric groups, or continuous groups, is a study of form. Also, the study of the construction of groups, whether by generators, or by the combination of groups, or in other ways, is also a study of structure or form. The calculus of operations in general, with such particular forms as differential operators, integral operators, difference operators, distributive operations in general, is for the most part a study of structure. In so far as any of these is concerned with the synthesis of compound forms from simple elements, it is to be classed as a study of form, as the term is here used.'[3]

In the notion of a group, we have become acquainted with two terms; namely, transformation and invariance. The first implies 'change'; the other, a lack of 'change' or 'permanence'. Both of these characteristics are semantically fundamental, but involve serious complexities.

The world, ourselves included, can be considered as processes which can be analysed in terms of transformed stages with all their derivative notions. In the objective world, 'change' is ever present and is, perhaps, the most important structural characteristic of our experience. But when a highly developed nervous system, a process itself, is acted upon by other processes, such nervous system discovers, at some stage of its development, a certain relative permanence, which, at a still later stage, is formulated as invariance of function and relations. The latter formulation is *non-el* because it can be discovered empirically, which means by the lower nerve centres, but also is the main necessity and means of operating of the higher nerve centres, so-called 'thought', . All that we usually call a process of 'association' is nothing else than a *process of relating,* a direct consequence of the structure of the nervous system, where stimuli are registered in a certain four-dimensional order, which, on the psycho-logical level, take the form of relations. From this point of view, it is natural that the higher nerve centres, as a limit of integrating processes, should produce *and* discover invariance of relations, which appears then as the supreme product and so, ultimately, a necessity of the activity of the higher centres. Obviously, if the invariance of relations has any objective counterpart whatsoever in the external world,

this invariance is impressed on the nervous system more than other characteristics; and so, at a certain stage, a nervous system which is capable of producing and using a highly developed symbolism, must discover and formulate this invariance.

It seems that *relations*, because of the possibility of discovering them and their invariance in *both* worlds, are, in a way, more 'objective' than so-called objects. We may have a science of 'invariance of relations', but we could not have a science of permanence of things; and the older doctrines of the permanence of our institutions must also be revised. Under modern conditions, which change rather rapidly nowadays, obviously, some relations between humans alter, and so the institutions must be revised. If we want *their invariance*, we must build them on such *invariant relations between humans* as are not altered by the transformations. This present work, indeed, is concerned with investigating such relations, and they are found in the *mechanism of time-binding*, which, once stated, becomes quite obvious after reflection.

As Professor Shaw says: 'We find in the invariants of mathematics a source of objective truth. So far as the creations of the mathematician fit the objects of nature, just so far must the inherent invariants point to objective reality. Indeed, much of the value of mathematics in its applications lies in the fact that its invariants have an objective meaning. When a geometric invariant vanishes, it points to a very definite character in the corresponding class of figures. When a physical invariant vanishes or has particular values, there must correspond to it physical facts. When a set of equations that represent physical phenomena have a set of invariants or covariants which they admit, then the physical phenomena have a corresponding character, and the physicist is forced to explain the law resulting. The unnoticed invariants of the electromagnetic equations have overturned physical theories, and have threatened philosophy. Consequently the importance of invariants cannot be too much magnified, from a practical point of view'.[4]

It should be noticed that the *non-el* character of the terms relation, invariance. , which apply both to 'senses' and 'mind', is particularly important, as it allows us to apply them to all processes; and that such a language is similar in structure not only to the world around us, but also to our nervous processes. Thus, a process of being iron, or a rock, or a table, or you, or me, may be considered, for practical purposes, as a temporal and average invariance of function on the sub-microscopic level. Under the action of other processes, the process becomes structurally transformed into different relational complexes, and we die, and a table or rock turns into dust, and so the invariance of this function vanishes.

The notion of a function involves the notion of a variable. The functional notion has been extended to the propositional function and, finally, to the doctrinal function and system-function. The term transformation is closely related to that of function and relation. This notion is based on our capacity to associate, or relate, any two or more 'mental' entities. We can, for instance, associate *a* with *b* or *b* with *a*. We say that we have transformed *a* into *b*, or vice versa.

An excellent example of transformation, given by Keyser, is an ordinary dictionary, which would be genuinely mathematical if it were more precise. In a dictionary, every word is transformed into its verbal meaning, and vice versa. A telephone directory is another example. Quite obviously, the term 'transformation' has far-reaching implications. If *a* is transformed into *b*, this implies that there is a relation between *a* and *b* which is being established, by the fact of transformation. Once a relation is established, we have a propositional function of two or more variables which define an extensional set of all elements connected by this relation.[5]

We see that these three terms are inseparably united and are three aspects of one psycho-logical process. If we have a transformation, we have a function and a relation; if we have a function, we have a relation and a transformation; if we have a relation, we have a transformation and a function. Transformation, as we see, is a psycho-logical term of action. A relation has a psycho-logically mixed character. A propositional function is a static statement, on record, with blanks for the values of the variables. In it the form is invariant, but it may take an indefinite number of values. The *extensional manifold of* the values for the variable is static, given once for all in a given context. It is extensional and, therefore, may be empirical and experimental.

Let us take as an example, for instance, the transformation of a set of integers 1, 2, 3, . Let us suppose that the given law of transformation is given by the function $y=2x$. The result would be the manifold of even integers 2, 4, 6, . We see that integers are transformed into integers; therefore, the characteristic of being an integer is preserved; in other words, this characteristic is an invariant under the given transformation $y=2x$, but the values of the integers are not preserved.

The theory of invariance is an important branch of mathematics, made famous of late through the work of Einstein. Einstein fulfilled the dearest dream of Riemann and attained the methodological and scientific ideal, that a 'law of nature' should be formulated in such a manner as to be invariant under groups of transformations. Such a semantic ideal, once stated, cannot be denied; it expresses exactly a necessity of

the proper working of the human nervous system. In fact, a ‘law’ of nature represents nothing else than a statement of the invariance of some relations. When the Einstein criterion is applied, it renders most of the old ‘natural laws’ invalid, as they cannot stand the test of invariance. The older ‘universal laws’ then appear as local private gossips, true for one observer and false for another.

The method of the theory of invariance gives us the trend of relations that abide, and so expresses important psycho-logical characteristics of the human ‘mind’. Its further significance is revealed by Keyser in the suggestion that when a group of transformations leaves some specified psycho-logical activity invariant, it defines perfectly some actual or potential branch of science, some actual or potential doctrine.[6]

We all know how deeply rooted in us is the feeling, the longing for stability, how worried we are when things become unstable. Worries and fear are destructive to semantic health and should be taken into account in a theory of sanity. A similar semantic urge apparently moved mathematicians when they worked out the theory of invariance; it was a formulation of a necessity of the activities of the human nervous system. That similar semantic methods, if applied, would give similar results in our daily lives, scarcely needs to be emphasized.

We have already spoken of the mathematical theory of invariance as a mathematical species of a semantic theory of universal agreement. Similarly, in a $\overline{A}$-system based on relations and structure, it is possible to formulate a theory of universal agreement which would be structurally impossible in the *A*-system, and so the dreams of Leibnitz become a sober reality; but we must first re-educate our *s.r*.

Section D. Similarity in structure of mathematics and of our nervous system.

In the chapter on the Semantics of the Differential Calculus, the fundamental notions and method of this calculus are explained. Here we may say, briefly, that it consists in stratifying, or expanding into a series, of an interval of any sort which proceeded by large steps. The large steps are divided into a great number of smaller and smaller steps, which, in the limit, when the numbers of steps become infinite, take on the aspect of ‘continuity’ so that we can study the ‘rate of change’. When ‘time’ is taken into consideration, the dynamic may be translated into static, and vice versa; processes can be analysed at any stage,. This short description is far from exact or exhaustive; I emphasize only in an intuitive way what is of main semantic importance for our purpose.

The main object of the present chapter is to explain that the structure of the human nervous system is such that, on some levels, we produce dynamic abstractions; on others, static. As the organism works as-a-whole, for its optimum working, and, therefore, for sanity, we need a language, a method, which may be translated into a *s.r* by which to translate the dynamic into the static, and vice versa; and such a language, such a method, is produced and supplied by mathematicians. To some readers, these remarks may appear so obvious as to make it unnecessary to write them, but I have found, through personal observation of reactions of different individuals, and by a careful survey of the literature of the subject, that even many mathematicians and physicists do not have this *s.r* in all problems—or, at least, they do not know how to apply it.

In Part VII, elementary $\overline{A}$ methods are worked out, which supply the neurological semantic benefits of the calculus, very easily imparted to even small children *without any mathematical technique*, and establishing in them a mathematical attitude toward all language in general, training them in the only structural psycho-logics of sanity; namely, that of the calculus, which thus becomes the foundation of healthy and normal human *s.r*. And this, let us repeat again, without any mathematical technique. We find, also, that there are simple and *physiological* means, based on structure, of training our *s.r* and imparting the feel for the structural stratification inherent in the consciousness of abstracting.

To start with, let me mention briefly a quite unexpected, unconscious, structural *biological* characteristic of mathematics; namely, its (in the main) *non-el,* organism-as-a-whole character.

From the time of Aristotle, biologists, physiologists, neurologists, ‘psychologists’, psychiatrists and others have spoken a great deal about the organism-as-a-whole; yet, they have not seemed to realize that if they produce *el* terms, they cannot apply the *non-el* principle.

It will probably not be an exaggeration to say that the majority of mathematicians have never heard of this principle, and that, if they have, they paid no attention to it; *yet*, in practice, they have applied it very thoroughly. The main mathematical terms are *non-el,* organism-as-a-whole terms which apply to ‘senses’ as well as to ‘mind’. For instance, relation, order, difference, variable, function, transformation, invariance. , can mostly be seen as well as ‘thought’ of. The use of such terms prevents our speculation from degenerating into purely *el* speculations on words, a process always closely related to the morbid semantic manifestations of the ‘mentally’ ill, and obviously based on the pathological confusion of orders of abstractions, involving inappropriate evaluation.

This fact alone is of serious importance, as it indicates that mathematics is a language of similar structure to the structure of organisms and is a correct language, not only neurologically, but also *biologically*. This characteristic of mathematics, quite unexpectedly discovered, made the fusion of geometry and physics possible. It underlies, also, the theory of space-time and the Einstein theory. It will be seen later that it has also serious psycho-neurological importance.

It was already emphasized that the existing 'psychologies' are animalistic or metaphysical, because either they disregard one of the most unique human characteristics, such as the behaviour called mathematizing, or they indulge in speculations on, and in, *el* terms. It was suggested that no *human* 'psychologist' can actually perform his official task unless he is an equipped student of mathematics. Unless we actually apply the *non-el* principle, and take into account that the structure of languages introduces implications, unconscious in the main, and that no man is ever free from some doctrines and some so-called 'logical' processes involving physiological and semantic concomitants, no general theory of *human* 'psychology' can be produced.

The above solves a very knotty semantic problem, for we see that if we apply the *non-el* principle, any 'psychology' on the human level must become *psycho-logics*, though the old term 'psychology' could be retained as applying to animal researches only. The very name 'psychology', or the 'theory or science of mind', is obviously *el*, and treats 'mind' as an objective separate entity. As these results were originally reached independently, it is interesting to notice that the modern methods and the application of the structural positive knowledge 1933 lead to very many analogies and similarities, though this, after all, might be expected.

Notice the hyphen which, out of the *el* and delusional objectified 'space' and 'time', made the einsteinian space-time a language of *non-el* structure similar to the world around us; and the hyphen which out of *el* 'psychology' makes a *non-el* human discipline of psycho-logics. It seems that a little dash here and there may be of serious semantic importance when we deal with symbolism.

To facilitate exposition, it is useful to stress, in the present section, the neurological and psychiatrical side, as an outline of the methods of the calculus, and related subjects will, of necessity, require separate treatment.

When rats are trained to perform a simple experiment requiring some 'mentality' and afterwards a large part of the cerebral cortex is removed, their training may be wholly lost. If such decorticated rats are trained again, they re-acquire the habit as readily as before. It appears

that, with rats, the cortex is not essential for these learning processes. They ‘learn’ as well, or nearly as well, with their sub-cortical and thalamic regions.[7] In what follows, to avoid misstatements, I will use the rather vague term, yet sufficient for my purpose, ‘thalamic region’ or ‘lower centres’ instead of more specific terms, the use of which would complicate the exposition unnecessarily. With dogs, apes, and men, the situation is increasingly different. Their nervous systems are more differentiated. Their functional interchangeability is impaired. In the most complex human brain there still exists some interchangeability of function. When an arm, for instance, is paralysed through a brain-lesion, the arm may re-acquire a nearly normal function, though there is no regeneration of the destroyed brain tissue. However, the interchangeability is less pronounced than in the lower brains. There seems to be ho doubt that the thalamic regions are not only a vestibule through which all impulses from the receptors have to pass in order to reach the cortex, but also that the affective characteristics are strictly connected with processes. in these regions. It seems that some very primitive and simple associations can be carried on by the thalamic regions.

The cortex receives its material as elaborated by the thalamus. The abstractions of the cortex are abstractions from abstractions and so ought to be called abstractions of higher order. In neurology, similarly, the neurons first excited are called of ‘first order’; and the succeeding members of the series are called neurons of the ‘second order’, . Such terminology is structurally similar to the inherent structure and function of the nervous system. The receptors are in direct contact with the out-side world and convey their excitation and nerve currents to the lower nerve centres, where these impulses are further elaborated and then abstracted by the higher centres.

According to our daily experience and scientific knowledge, the outside world is an ever-changing chain of events, a kind of flux; and, naturally, those nerve centres in closest contact with the outside world must react in a shifting way. These reactions are easily moved one way or another, as in our ‘emotions’, ‘affective moods’, ‘attention’, ‘concentration’, ‘evaluation’, and other such semantic responses. In these processes, some associative or relational circuits exist, and there may be some very low kind of ‘thinking’ on this level. Birds have a well-developed, or, perhaps, over-developed, thalamus but under-developed and poor cortex, which may be connected with their stupidity and excitability.

Something similar could be said about the ‘thalamic thinking’ in humans; those individuals who overwork their thalamus and use their cortex too little are ‘emotional’ and stupid. This statement is not exag-

gerated, because there are experimental data to show how through a psycho-neural training the *s.r*, in some cases, can be re-educated, and that with the elimination of the semantic disturbances there is a marked development of poise, balance, and a proportional increase of critical judgement, and so 'intelligence'. Idiots, imbeciles, and morons are usually 'emotional' and excitable, as well as deficient in their 'mental' processes. A similar characteristic can be found in other unclassified 'mentally' deficient, and their name is legion—a characteristic strictly connected with, and often produced by, disturbances of the *s.r.* When these shifting, dynamic, affective, thalamic-region, lower order abstractions are abstracted again by the higher centres, these new abstractions are further removed from the outside world and must be somehow different.

In fact, they *are* different; and one of the most characteristic differences is that they have *lost* their *shifting* character. These new abstractions are relatively static. It is true that one may be supplanted by another, but they do not change. In this fact lies the tremendous value and danger of this mechanism, as disclosed clearly by the disturbances of the *s.r*. The value is chiefly in the fact that such higher order abstractions represent a perfected kind of memory, which can be recalled exactly in the form as it was originally produced. For instance, the circle, *defined* as the locus of points in a plane at equal distance from a given point called the centre, remains permanent as long as we wish to use this definition. We can, therefore, recall it perfectly, analyse it. , without losing the definiteness and the stability of this memory. Thus, critical analysis, and, therefore, progress, becomes possible. Compare this perfected memory, which may last indefinitely unchanged, with memories of 'emotions' which, whether dim or clear, are always distorted. We see that the first are reliable, that the others are not.

Another most important characteristic of the higher order abstractions is that, although of neural origin, they may be preserved and used over and over again in extra-neural forms, as recorded in books and otherwise. This fact is never fully appreciated from a neurological point of view. Neural products are stored up or preserved in extra-neural form, and they can be put back in the nervous system *as active neural processes.* The above represents a fundamental mechanism of time-binding which becomes overwhelmingly important, provided we discover the physiological mechanism of regulating the *s.r*, on the one hand, and discover the mechanism by which these extra-neural factors can be made physiologically effective, on the other.

If humans are characterized by the fact that they build up this cumulative affair called 'civilization', this is possible through those higher

order abstractions and the time-binding ability to extend our nervous system by extra-neural means, which, in the meantime, may play a most important neural role and become active nervous impulses. The last is only possible if some abstractions are static, and so can be recorded, leading ultimately to further extensions of the human nervous system by extra-neural means, such as microscopes, telescopes, and practically all modern scientific instruments, books, and other records.

To illustrate what has been said here, I know of no better example than is found in moving pictures. When we watch a moving picture representing some life occurrence, our ‘emotions’ are aroused, we ‘live through’, the drama; but the details, in the main, are blurred, and a short time after seeing it either we forget it all or in parts, or our memory falsifies most effectively what was seen. It is easy to verify the above experimentally by seeing one picture twice or three times, with an interval of a few days between each seeing. The picture was ‘moving’, all was changing, shifting, dynamic, similar to the world *and* our feelings on the un-speakable levels. The impressions were vague, shifting, non-lasting, and what was left of it was mostly coloured by the individual mood. , while seeing the moving picture. Naturally, under such conditions, there is little possibility of a rational scientific analysis of a situation.

But if we *stop* the moving film which ran, say, thirty minutes, and analyse the static and extensional series of small pictures on the reel, we find that the drama which so stirred our ‘emotions’ in its moving aspect becomes a series of slightly different static pictures, each difference between the given jerk or grimace being a *measurable* entity, establishing relations which last indefinitely.

The *moving* picture represents the usually brief processes going on in the lower nerve centres, ‘close to life’, but unreliable and evading scrutiny. The *arrested* static film which lasts indefinitely, giving *measurable* differences between the recorded jerks and grimaces, obviously allows analysis and gives a good analogy of the working of higher nerve centres, disclosing also that all life occurrences have many aspects, the selection of which is mostly a problem of our pleasure and of the selection of language. The moving picture gives us the process; each static film of the reel gives us stages of the process in chosen intervals. In case we want a moving picture of a growing plant, for instance, we photograph it at given intervals and then run it in a moving-picture projector, and then we see the process of growth. These are empirical facts, and the calculus supplies us with a language of similar structure with many other important consequences.

It is characteristic that those who claim to be most interested in human affairs and human processes, whom we call, among others, 'philosophers', 'psychologists'. , should not have discovered much of value in these fields. But mathematicians, who disclaim meaning in their undefined terms, or 'truth' in their postulates, or interest in human affairs, have had a most astonishing and unique success by elaborating methods for the translation of the dynamic into static and the static into dynamic. Claims and disclaims matter little, but working in accordance with the survival order of the nerve structure and currents has produced most valuable results.

The different methods of mathematics and the four-dimensional 'world' of Minkowski form the means for translating the dynamic into static and vice versa. Minkowski established a language of a new structure, closer to actual facts of the world around us and ourselves, making the general theory of Einstein possible. Further analysis of these issues is carried out in Part IX, and it is one of the semantic foundations upon which a positive theory of sanity can be built.

Disclaiming definite meanings, mathematicians have an intuitive predilection for selecting their terms and pursuing their line of enquiry among *possible meanings*, although formally these meanings are disregarded. The feeling which directs the selection of material which is formally interesting and important is akin to the artistic sense, but, unfortunately, in spite of its importance, it has been neglected by 'psychologists'. Quite often it is the 'feel' which directs the mathematicians in their researches and suggests or modifies lines of development or the selection of one set of postulates in preference to other sets. This is why the ordinary sense of the terms used in mathematics is so important, although it represents only some of the possible meanings. These, with their implications, usually represent most important structural characteristics of the human nervous system and the world.

This is to be expected because of the reasons given above; the more so that invariance in this shifting world is a characteristic of relations, and mathematics is a language of exact relations which, in the meantime, have mostly objective counterparts. The highest abstractions at every date are detached from the outside world neurologically, and *should remain detached,* to represent 'pure mind' in action. These higher abstractions are on the public level, as they are transmittable verbally with all characteristics included. They are static, unhampered directly by the outside events, although they normally originate in them. These higher order abstractions are 'digested' and translated into lower order abstractions and returned to the lower centres; and they receive their

meanings close to life. Such meanings are enlightened meanings, a survival process, and each nervous level did its work properly.

We know that a number of human races have perished without leaving many traces of their existence. This process is going on continually, even now. Some races are progressing; some are regressing; some are at a seeming standstill. It would appear that the mechanism of higher order abstractions had and has survival value, and, therefore, should not be neglected but cultivated. In this special case, cultivation is a condition inherent in the process and a necessity for time-binders.

Serious semantic dangers are also revealed by analysis and verified by observation. These higher order abstractions, let us repeat, are static and may last indefinitely, as long as for structural reasons we do not replace the old by new ones. Even then, though rejected, they remain as a permanent fact on record. Obviously, these higher abstractions have only a 'second-hand' connection with the outside world. Even their character is changed, they are static while the world is dynamic. The lower 'sense' world has 'characteristics left out', owing to the mechanism of abstracting of the lower centres; and the abstractions of higher orders have 'all characteristics included', because these are abstractions from abstractions, an *intra-organismal* process in its entirety, their starting material being already an end-product of the activities of the lower centres. This mechanism is only under full control if we are conscious of abstracting, because the higher order abstractions in the nervous chain affect, in their turn, the lower centres, and, in pathological cases, impress on them a semantic *delusional* or *illusional* evaluation as if a character of experience. In severe cases, even the lower nerve centres are stimulated to such an extent that hallucinations appear.

If we do not know how to handle different order abstractions, this results in serious semantic dangers. If the distribution of the returning nerve currents is a non-survival one, we exhibit semantic disturbances, such as identification or confusion of orders of abstractions, delusions, illusions, and hallucinations. Thus, we ascribe to the products of the lower nerve centres, the lower order abstractions, characteristics fictitious and impossible for them, such as 'immutability', 'permanence', involving disorientation about 'time'. , . , which are characteristics of the higher order abstractions, but do not belong to the world as given by the lower abstractions, and result in an improper evaluation disturbing to the *s.r.* Such disturbances make us, naturally, absolutists and dogmatists, involve serious affective disturbances, and lead to non-adaptive behaviour and reactions, and other semantic manifestations of un-sanity. These, in their turn, make adjustment more difficult, often affecting the

structure of man-made institutions, which again make adjustments more complex and often impossible. We become un-sane, 'insane', and life, whether public or private, becomes a mess. In such a vicious semantic circle, we distort our education, our systems, and institutions. Often the morbid reactions of powerful individuals are forced upon masses, who are then ruled by these morbid products, with injury to their nervous systems. Different mass hysterias, 'revivals', wars, political and religious propaganda, very often commercial advertisements, offer notable examples.

The morbid semantic influence of commercialism has not been investigated, but it does not take much imagination to see that commercial psycho-logics, as exemplified by the theories of commercial evaluation, 'wisdom', appeal to selfishness, animal cunning, concealing of true facts, appeal to 'sense' gratification. , produce a *verbal and semantic environment* and slogans for the children which, if preserved in the grown-ups, must produce some pathological results. It is hoped that some day a psychiatrist will investigate this large, neglected, and very important semantic problem.

The lack of structural linguistic researches and investigation of our *s.r*, and the ignorance of those who rule, make us nearly helpless. Malaria or other germ diseases would never be eliminated were we to preserve religiously the sources of infection. The semantic sources of un-sanity are not only defended but are actively sponsored by organized ignorance and the power of merchants, state, and church.

The situation is acute. If we could entirely eliminate our cortex, it would, perhaps, not be so serious. We could, perhaps, live as complex a life as a fish and have a nervous system perfectly adjusted to such a life. But, unfortunately, with a structural change, or, according to Lashley, with the change even in the total mass of the brain, the activities and the role of the whole, including other parts, are profoundly altered.[8] These become inadequate, as shown by the boy born without the cortex, already described. His nervous system was much more complex than that of fishes or of some lower animals which lead *adequately* a rather complex life. But the boy was less equipped for life than they. Even his 'senses', though apparently 'normal' on macroscopic levels, must have been pathological on colloidal and sub-microscopic levels and did not function properly. We know, also, that in many cases of 'mental' ills the 'sense reactions' are abnormal; sometimes the patients seem to be entirely insensitive to stimuli which would produce most acute pain to other less pathological individuals.

It is impossible to eliminate completely from our lives or nerve currents the higher abstractions and their psycho-neural effect. Curiously enough, this elementary fact has never been emphasized or taken into account seriously; yet it is a crucial semantic factor in our attitude toward science and our future. Those who attempt such elimination, whether by actively persecuting science, or by emitting propaganda against science, or by the cynical or ignoring attitude toward 'mental' achievements, whether personally, or in education, or in public prints, or other public activities, do not succeed in eliminating the higher order abstractions, but simply introduce *pathological semantic reactions* and succeed in disorganizing their own nervous systems and those of others. I intended this implication when I said that our existing educational. , systems *produce* morons, but 'geniuses' are born. Such very general semantic directives are, perhaps, responsible for the extremely low level of our non-technical development. Humans are not to be judged simply by the ability to drive an automobile or by the knowledge of how to use a bathtub; nor yet by their capacity for buying and selling things produced by others.

The tendency of some public prints to appeal to the morbidity of mob psycho-logics and to its ignorance, insisting that all that is said should be said in 'one-syllable' words, so that the mob can understand, in a human class of life, is an *arresting* or *regressive tendency.* What should be urged for sanity, and for humans, is that the mob should also learn the use of at least two-syllable words ! Then, perhaps, the day would come when they could follow easily and habitually the use of *non-el* terms and, perhaps, even of words connected by a hyphen.

This appeal to mob psycho-logics and ignorance affects profoundly our *s.r* and should be investigated. It definitely appears that in countries where the majority reads only the sort of publications referred to above and commercial advertisements, their psycho-logical equipment and standards are lower than those of perfectly illiterate peasants of other countries. It is not fully realized that in a symbolic class of life, symbolism of any sort—e.g., public prints—plays an environmental role and creates *s.r* which may be distinctly morbid. The problems of public prints, commercialism. , and their psycho-logical effect on the *s.r* should undergo a searching analysis by psychiatrists, and definite suggestions should be formulated by psychiatric scientific organizations or congresses.

Under the conditions prevailing at present, it is futile to preach 'morals' of any metaphysical kind. They have never worked satisfactorily, and increasingly they cannot work, particularly under the present much more complex conditions of life. They disorganize the survival

activities and processes of the human nervous system. The imposed and delusional dogmas are themselves the result of pathological evaluation in their originators; a necessity, perhaps, on a primitive level, but profoundly semantically harmful under the complexities of life-conditions 1933.

As it is impossible to eliminate the influence of the higher order abstractions, we should investigate whether or not we can control these processes and the related *s.r*. We can learn to regulate these processes, which otherwise may become pathological, and to redirect the currents into constructive survival channels. I can state definitely that this is possible. We can control physiologically the *s.r* through the elimination of identification, by training in order, in consciousness of abstracting, and similar disciplines, and thus eliminate the pathological semantic disturbances of confusion of orders of abstractions. Such training, whenever possible, has seemingly a beneficial influence even on the more extreme pathological states listed above, and suggests general preventive value.

Let me briefly restate the fundamental differences between lower order abstractions and higher. The lower order abstractions are manufactured by the lower nerve centres, which are closer to, and in direct contact with, actual life experiences. These are non-permanent, shifting, vague and un-speakable, but often very intense. They play a most important role in our daily lives. They cannot be transmitted, as they are essentially of a non-transmittable character, and have a private, non-public character. All ‘sense’ impressions, ‘feelings’, ‘moods’. , are representative. of them. We should remember that, detached, they are fictions, manufactured verbally, because our language happens to be *el*. Actually, these lower centres are in the cyclic chain and so influence, and are influenced by, the full cycle, including the higher order abstractions, whatever the latter may be in a given individual. The main point is that they are shifting, changing, non-permanent, non-stable—’moving’, so to say—and remain un-speakable.

The higher order abstractions are abstractions from the lower order abstractions, being further removed from the outside world, and are of a distinctly different character. These are static, ‘permanent’, and cannot be entirely eliminated from any one.

From the point of view of sanity, the problem of how we can handle these functions becomes paramount. In the cyclic nervous chain, we always must translate one level into the other. Obviously, if, in the *higher* centres, we elaborate shifting, changing, non-permanent material, this material is not appropriate for them; they cannot work properly, and some pathological processes may set in.

If we elaborate the *lower* nerve centres abstractions that are static, permanent. , in character, and hence inappropriate for the lower centres, we build up morbid non-survival identifications, delusions, illusions, hallucinations, and other disturbances of evaluation, resulting in milder cases in absolutism, dogmatism, fanaticism. , and, in heavier cases, in a neurosis or even a psychosis.

It seems quite obvious that each nervous level has its own specific kind of material to deal with. As they are in a cyclic nervous chain and are interconnected in a bewilderingly complex way, the problem of appropriate translation of one level of abstractions into the other becomes a semantic foundation for a well-balanced functioning of the nervous system. In this respect, we differ fundamentally from animals. The above difficulties do not arise in animals to that extent, because their nervous systems are not differentiated enough for such sharp differentiation in the functioning. For this reason, without human interference, there could be no 'insane' animals which could survive (see Part VI). But, having no static higher order abstractions in the human sense, they cannot pass on their 'experiences', which are transmittable *only* in the higher order formulations in neural and extra-neural forms to the next generations. Animals are not time-binders.

For humans, the proper translation of dynamic into static and static into dynamic becomes paramount for sanity, on psycho-logical levels, affecting, probably by colloidal processes, the psycho-neural foundation of semantic responses.

Psychiatry informs us that most of the 'mentally' ill have their main disturbances in the dynamic affective field. It is a very difficult field to reach by the older methods, the more so that the older *el* sharp distinction between 'intellect' and 'emotions' prevented the discovery of workable means. 'Thinking' and 'feeling' are not to be divided so simply. We know how 'thinking' is influenced by 'feeling'; but we know very little how 'feeling' is influenced by 'thinking'—perhaps, because we have not analysed the semantic issues in *non-el* terms.

All psychotherapy, with its manifold theories, each contributing its share, is a semantic attempt to influence 'feeling' by 'thinking'. A large number of successful cases seems to show clearly that some such means are possible. Large numbers of failures show equally that the methods used are not structurally satisfactory. The need of more scientific investigations of a more general and fundamental, *non-el* character becomes emphatic. The present enquiry shows that such structural investigations suggest that the method can be found in the psycho-logics of the 'mind' at its best; namely, in mathematics, which unexpectedly leads to a

physiological control of the *s.r*, effective not only as a therapeutic, but also as a preventive, educational means.

Identification as a factor of un-sanity seems to be a natural consequence of the evolution from ‘animal’ to ‘man’, particularly at our present stage, while the human race is so recent a product. The human cortex appeared only comparatively lately and is a young structure; the thalamic regions have a much longer history of functioning. It seems natural that the nervous impulses should pass the shorter, more phylogenetically travelled, paths in preference to comparatively newer and longer paths, a principle well known in neurology in connection with so-called ‘Bahnung’. If education, and on human levels any kind of adjustment involving *s.r* involves some education, fails to force the nerve currents into their proper channels, or actively establishes in them semantic psycho-neural blockages through pathological evaluation acquired because of faulty training, we should expect either infantilism or regression to still lower levels. Whatever the correct explanation of the distribution of nerve currents, semantic blockages. , may be, observation shows unmistakably that some such assumptions are necessitated by observed manifestations in behaviour. Experiments show, also, that such defects can be helped greatly by the proper re-training and re-education of the *s.r*.

To understand the structure of these semantic disturbances, we must become acquainted with the affective components which underlie mathematics and mathematical methods, hitherto disregarded, because of the *el* character of our old terminology. There is another striking connection. In severe ‘mental’ illnesses, we usually find a disorientation in ‘space’ and ‘time’, which are, by necessity, *relational data* of experience. In the semantic disturbances called identification, we also find, as a rule, relational disorientation *about* ‘space’ and ‘time’, more subtle but very vicious in effect, bordering on what are called ‘philosophical’ problems, which, as a matter of fact, represent psycho-neural disturbances. Since Einstein, the disturbances can be easily eliminated, provided we take into account structural *non-el* issues in connection with *s.r* and a $\overline{A}$-system.

It is instructive to make a short survey of the methods by which the mechanism of the nervous cycle—’senses’, ‘feelings’. , first; ‘mind’, which again influences the ‘feelings’, next—works in mathematics. Weierstrass, the famous mathematician, says, in one of his writings, that a mathematician is a kind of poet. This is largely true. Mathematics is not only a rigorous linguistic relational pattern, but it uses the highest abstractions which we have reached at a given period from

the data given by the lower nerve centres, which are closer to experience, or rather which constitute experience. The older arguments about the connection or lack of connection between the lower order abstractions (‘sense’ data. ,) , and mathematics are due solely to a confusion of orders of abstractions and are a useless gambling in *el* terms. Only in severe ‘mental’ ills is the speech of the patients entirely unconnected with first order external ‘realities’, and so the study of relations of many kinds and orders, called ‘mathematics’, cannot, as long as it is sane, be entirely detached from ‘reality’. In fact, it is useless for mathematicians to try to produce disciplines which have no practical applications. As long as it is professionally accepted as mathematics, and, therefore, a science and sane, whatever mathematicians produce will always be connected with lower order abstractions, and must have an application sooner or later. When these higher order abstractions, produced very often by many individuals, are absorbed and returned in a modified form to the lower centres as ‘visualization’, ‘intuition’, ‘feelings’. , the given individual is closer to the external world than he was before, because he has absorbed, digested, and appropriated the nervous results of many more experiences than he himself could have gathered alone. He is able to compare, evaluate, and relate, revise and adjust his private experiences and observations with the *translated* experiences from higher abstractions of many more individuals. The *translation* is indispensable, because the reactions of both levels are entirely different, and comparable only when they are on one level. *Creative work has begun.*

Experiences given by the lower centres and lower abstractions are full of meanings, colouring, affective and semantic components, and these are not directly comparable with the higher abstractions produced by the higher nerve centres. They must be first transformed, ‘digested’, and translated into terms of the lower centres, which are the only ones which are effective on the lower levels. We call them ‘visualization’, ‘intuition’, ‘feeling’, ‘culture’, . The exact mechanism is not well known, but we have a number of data which show that the lower nerve centres are somehow engaged in these processes.

When this is accomplished, the mathematician has at his disposal an enormous amount of data; first, his personal experiences and observation of actual life (lower centres and lower order abstractions), and also all the personal experiences and observations of past generations. Although the latter were stored in the form of higher order abstractions only as an *account* of past experiences in neural or extra-neural forms, his nervous cycle was affected by them, and they were translated back into experiences of the lower levels.

With such an enormous amount of data of experience, he can *re-evaluate* the data, ‘see’ them anew, and so produce new and more useful and structurally more correct higher order abstractions. In their turn, these will produce similar semantic effects with other individuals, . The mechanism is, after all, well known and general, obvious even in the relations between some feeble-minded parents and their eventually feeble-minded children. It is entirely obvious on racial grounds; but, at present, it is not so obvious, and often but slightly effective, on personal and individual grounds, because we have had no means of training structurally and effectively the *s.r* in proper evaluation. The mechanism is entirely general, but it is obvious and seen at work in the majority of *creative* scientists and so-called ‘geniuses’. These processes have not been analysed in terms of order, and so, although we use them often, we are not conscious of their mechanism and have no means of training our *s.r*. The *s.r* are a product of training, education. , and are not inborn in *a given form.* Even birds bred in a laboratory which have never heard their parents or other birds sing will sing, as this is an inborn reflex, but the melody produced is different from that of their parents. Under normal conditions, the form of the song is standardized and is a result of *copying* parents. In other words, the melody-environment has affected them. With humans, it is not only a question of the given noises, the ‘melody-environment’ which we relate with some experiences, but the *s.r* involve affective responses to meanings, and this depends on the structure of language, involving unconscious, yet vital, evaluation factors and our *attitude* toward language, which ultimately depends on our knowledge of the mechanism and use of language.

These problems are extremely complex and subtle, and, at this stage, we are not ready to go into further details, the more so that there is a very simple and effective physiological structural method given in Part VII, which in practice eliminates enormous theoretical difficulties. There is little doubt that this mechanism of recasting, or translation of abstractions, is present in all of us, but this mechanism requires knowledge of the proper way to handle it, and that knowledge is not inborn, but has to be acquired by education. Up to the present date, these problems have been disregarded, and the *s.r* treated in a haphazard way; once the physiological mechanism of these reactions is discovered, however, we shall be able to use its benefits without the inherent dangers of disturbances.

Here we must face a rather unexpected fact.

Mathematics is alone and unique in that it has no content or definite meanings ascribed to the undefined terms; and, therefore, only in mathe-

matics can we avoid the vicious influencing of lower centres through the feeling of false analogies which distort and disorganize the process. It is important to notice that the main and only lasting advances in 'philosophy' have been made by mathematicians; and, as a rule, whenever a trained mathematician attempts to work at any other profession not requiring mathematics, he shortly becomes an outstanding worker in the new field. It must be obvious that the returning nerve currents, when they produce the 'feel' (language of the lower centres) of physics, or chemistry, or biology, or other sciences with a definite content, must have a most pronounced semantic effect. Because of this physical content, identification and other semantic disturbances are usually present, instead of the highly beneficial visualization.

Empirically, this is quite obviously true. Let us survey the character of this process in physicists and chemists. Their problems, the content of their abstractions, are obviously not so closely related to human lives as the problems of biology. History shows that the attitude (affective) of those scientists toward human affairs is often shallow, but very seldom vicious or harmful. But let us take the attitudes of biologists, whose subject is seemingly much closer, or, at least, more affectively related to our problems, and we see, from Aristotle on, the brutalizing and *unscientific* (1933) effect of the false biological analogies. Practically all the vicious, unjustified, and unscientific generalizations which have made the white race the most animalistic, selfish, cruel, hypocritical, and un-sane race on earth are mainly due to the biological, *A*, distorted reasonings and *s.r* produced by false analogy.

In all this 'philosophy', they always reasoned from pigs, cats, and dogs to man. Since they were 'scientific', we blindly assumed that they must know what they were talking about. Even today, the majority of the older biologists refuse to investigate the structure of their language. They do not seem to be able to realize that most biological 'philosophies' are structurally fallacious and unscientific in 1933. They still unconsciously follow Aristotle. They refuse to understand that life is made up of absolute and *unique* individuals, and that 'man' or 'animal' *is not* an object, but labels verbal fictions.

In actual life, the differences between individuals are absolute, and father and son are different. These are the empirical facts of their sciences, the rest being verbal fictions. The notorious Tennessee trial demonstrated that in a large country like the United States of America, with a few good universities, there was no biologist to voice these points about 'evolution'. It is true that, through the work of neurologists and some others, biologists, of late, are beginning to see that they cannot

generalize in the way they have done for more than two thousand years. Naturally, there are notable exceptions; yet even these do not realize the structural linguistic and semantic issues involved.

I most emphatically do not deny that animal researches are extremely useful and necessary; but I question the right of biologists to remain innocent of the importance of linguistic and semantic issues, and to indulge in vicious, unwarranted generalizations which, although they may express their own metaphysics and *s.r*, should not be advanced as 'scientific' results. Biologists ought to be informed enough to understand that 'man' and 'animal' are verbal fictions, and labels for something going on inside our skins—not labels for the unique individuals with which they have to deal outside their skins.

An example may, perhaps, be useful. We know that rats, prairie dogs, and some other animals are mostly immune to scurvy, but that man, monkeys, and guinea pigs are mostly not immune. How can we generalize from a rat to a man *or* a guinea pig ? Or how much can we learn about the behaviour of a bee from the behaviour of an oyster, to use the example of Professor Jennings ? Even in 'man', what helps one 'man', kills another.

Similar false analogies occur in the *A* classification of 'man' as an 'animal'. This classification disregards completely the *s.r* and twists the generally accepted folk-meaning of the term 'animal' into a special meaning which introduces very vicious semantic implications. If we classify 'man' as an 'animal', the structural *A* 'plus' elementalism is automatically introduced, since 'man', obviously, has many characteristics of behaviour not shown by the 'animal', taken in its folk meaning. The disregard of the folk-meaning in our terminology shows clearly the complete disregard for *s.r* which are very strongly related to those folk-meanings. If we are to call 'man' an 'animal', then 'man' must be an 'animal' 'plus' something. If we were to call him some sort of a junior 'god', he would be a 'god' 'minus' something. The latter structural fallacy would be just as vicious in its implications, and would again deliver our speculations into the semantic clutches of the structure of a primitive-made *el* language.

Similar objections could be raised to that class of 'biological psychologies' exemplified by the 'behaviourists'—(not to be confused with the illuminating and highly constructive biological psychiatry or psychobiology introduced by Professor Adolf Meyer).[9] The 'behaviourists' try to be ultra-'scientific', not realizing that their knowledge of scientific method and structure belongs somewhere to the sixteenth century.

Creative mathematicians, after becoming acquainted with the work of their predecessors and contemporaries, achieve their own results, at first, through ‘intuition’, ‘feeling’, . They ‘visualize’ the most abstract theories, though sometimes it takes the invention of new means to achieve this result. Their lower nervous centres are affected by the higher abstractions made by themselves and others. This process accounts for the fact that no mathematical achievement is ever detached, or possibly can be detached, from life. The source of all creative work is always in the lower centres, which are in more direct contact with the world around us, through ‘feelings’, ‘intuitions’, ‘visualization’, and other first order reactions.

Mathematics and what is called ‘sublimation’ in psychiatry have a similar neural mechanism, which is expressed structurally in the spiral theory, or in the cyclic chain of nerve currents, where the end-product of one process becomes the starting point of the next. As was said before, this is quite obvious on racial grounds, but more difficult to discover or apply in individual experience, if we disregard structure and *non-el s.r.*

If we can, let us discover means by which the ‘feel’ of modern science can be imparted without falsification and technicalities, which, perhaps, may be only auxiliary means to get the more fundamental life results. We may at once anticipate the means which we shall discover. The key problem is to eliminate, first, the semantic disturbance called identification or the confusion of orders of abstractions, and similar disturbances of evaluation. This elimination is attained physiologically through the development of the consciousness of abstracting, which leads to proper evaluation, visualization *without* semantic disturbances. In other words, we must find means by which higher abstractions can be translated physiologically into lower abstractions, uniquely connected with the translation of the dynamic into the static and vice versa.

The present status of the white race—I do not know enough about the structures of languages of other races and their *s.r* to speak about them—is such that a majority of our self-imposed difficulties is due to the lack of scientific structural analysis, which lack makes it impossible to control or regulate physiologically and adequately the semantic evaluation through education. Under such conditions, everything based on arguments involving the ‘is’ of identity and the older *el* ‘logic’ and ‘psychology’, such as the prevailing doctrines, laws, institutions, systems. , cannot possibly be in full accordance with the structure of our nervous system. This, in turn, affects the latter and results in the prevailing private and public un-sanity. Hence, the unrest, unhappiness, nervous

strain, irritability, lack of wisdom and absence of balance, the instability of our institutions, the wars and revolutions, the increase of 'mental' ills, prostitution, criminality, commercialism as a creed, the inadequate standards of education, the low professional standards of lawyers, priests, politicians, physicians, teachers, parents, and even of scientists—which in the last-named field often lead to dogmatic and antisocial attitudes and lack of creativeness.

This is, naturally, an unsatisfactory semantic state of affairs, and, in consequence, our nervous systems do not function properly, according to the potentialities of proper evaluation inherent in their structure. False creeds or doctrines underlying the *s.r*, particularly when connected with strong affective tension, play as great a havoc with our responses and capacities on sub-microscopic colloidal levels as any macroscopic organic lesion of our nervous system. If our *s.r* are pathological, invariably some affective disturbance, and psycho-neural blockages on the colloidal level, must be present. The nervous currents are then deviated and forced into lower, non-survival-for-man channels, resulting in various forms of arrested development or regressive symptoms. Through this we are deprived of the higher (*human*) 'intelligence', which is the result of the optimum working of the nervous system on all levels; we become 'mentally' deficient in various aspects and degrees, and we have to copy animals, primitives, and infants, and so present, in milder disturbances, the pathetic picture—so often seen—of adult infantilism, or display other regressive manifestations. Thousands of such cases have been analysed and recorded in psychiatrical literature. The mechanism of these disturbances is quite clear, because, after the re-education of the *s.r*, if this is at all successful, the psycho-neurological colloidal blockage is eliminated, and the patient is relieved from his semantic afflictions.

Instances of infantilism and animalistic reactions are abundant everywhere; but as this problem is analysed further in Part VII, here we shall not pursue the matter further.

It should be noticed, however, in this connection, that sex abnormalities of every description and most sex disturbances are also interconnected with infantilism in adults. In public life and activities, the results are equally pathetic. Instead of analysing and foreseeing, we proceed by trial and error, as animals do, a wasteful and painful method. The possession of an adequate physiological method for the translation from the appropriate reactions of one level to that of another, therefore, becomes paramount. The *non-el* language and the methods of mathematics appear, then, to be of neurological value. The terms are easily

and correctly applied to both levels, and thus facilitate passing from the language appropriate to one level to the language appropriate to the other. But, in this case, to avoid confusion, we should have to make clear the multiordinality of terms and to embody recognition of this multiordinality in every, even the most elementary, education, as any education shapes and moulds some *s.r*. This will aid the working of the human nervous system, which, at present, is blocked, sometimes very effectively, by disturbances of evaluation. The old *el*, subject-predicate language has a structure dissimilar to the structure of this world as we know it in 1933, and also dissimilar to the structure and function of the human nervous system, and so, by necessity, hampers the *s.r* and deviates them from their natural course.

That the problems before us are subtle, and that the demarcation line between 'sanity', 'un-sanity', and 'insanity' is extremely thin, is no reason for neglecting this neurological benefit of psychophysiological investigation. It seems obvious that the attitude toward our forms of representation, and toward our *s.r*, are fundamentally affected by the disturbances of evaluation called identification or confusion of orders of abstractions, and, in particular, by objectification, which ascribe unjustified and delusional values and meanings to these forms.

Up to this point, we have been emphasizing the beneficial structural aspect of mathematics, and it is now necessary to explain why mathematizing, when considered as a formal interplay of contentless symbols should not be considered a high-class 'mental' activity, no matter how useful and important it may be, and why the majority of mathematicians do not get the *full* psycho-logical semantic benefit of their training and activities. The nervous systems of many such mathematicians do not act fully and successfully, nor pass normally through the cycle of their natural activities. Such a technician is seldom, if ever, what we call a great man. He seldom has a direct creative influence on our lives. But, in the case of a man with a more efficient nervous system, the cycle is completed successfully, the higher abstractions are translated back into new lower abstractions, which are closer to life. Such an individual 'sees', 'visualizes', has 'intuitions'. , in his symbolic interplays. He then has a new structural vision through a new survey of his own experiences and all the experiences of others when translated in terms of lower centres. He gains a deeper insight, which he ultimately makes useful to all of us.

Immediate experience, always un-speakable, is strictly connected with the lower centres. In the translation of experience into higher order abstractions and language, the un-speakable character of experi-

ence is lost, and a *new neurological process* is needed to re-translate these higher order abstractions into new lower abstractions, and thus fully and successfully to complete the nervous cycle. One can learn to play with symbols according to rules, but such play has little creative value. If the translation is made into the language of lower centres—namely, into ‘intuitions’, ‘feelings’, ‘visualizations’. ,—the higher abstractions gain the character of experience, and so creative activity begins. Individuals with thoroughly efficient nervous systems become what we call ‘geniuses’. They create new values by inventions of new methods and in other ways, which give us a new structural means of exploring, and thus of dealing with, the world around us and ourselves, and so, ultimately, human adjustment is helped.

It is important for the reader to become thoroughly familiar with the simple division of our nervous processes into terms of order in a cyclic chain. Even neurology calls the neurons excited first of ‘first order’, and the succeeding members of the series, of ‘second order’, . The above considerations have an important practical semantic bearing for all of us, since many of the processes which we are describing can be influenced educationally by simple methods, because the term ‘order’, when applied, acquires a *physiological* character for *evaluation.* The description and verbal analysis of the process is, naturally, complex, but once the physiological base of evaluation is discovered, the training becomes very simple, although not easy.

The principal aim of this present work is to make available a simple and practical physiological means for accomplishing what is highly desirable, and, at the same time, for eliminating what is semantically undesirable. We deal with mathematics, because mathematics is *unique,* and, being unique, has no substitute. When discussing the theory of meanings, we have shown that all verbalism is, ultimately, similar to mathematics in structure. This conclusion contradicts many current theories of language and meanings, and so, at this stage of our argument, we lay special emphasis on the only discipline in which these issues are clear and obvious; namely, mathematics. The older theories, based on ignorance of mathematics, have led to serious abuses of our linguistic capacities and to *s.r* which are mostly pathological, with the result that practically 99 per cent of us are semantically disturbed and un-sane. Many of us, even, are on the verge of more serious ‘mental’ illnesses.

It will be well to give a rough picture of the similarities of, and differences between, the working of the human ‘mind’ at its worst (‘insanity’), and its working at its best (mathematics). We shall find that the average man is between the two, often dangerously close to the

first. The following picture is rough and one-sided, but suggestive, and should be worked out more fully.

The ‘insane’ have structural, conscious or unconscious, ‘premises’, which are ‘false’, or, in general, semantically inappropriate. Their *s.r* are shifting when they should be static, or static when they should be flexible. In the main, the difficulty of evaluation lies in the lower abstractions and the affective field. These abstractions are not properly transmitted or translated or regulated by the higher centres; or else, the higher order static abstractions are projected with too strong affective components on the lower centres. Hence, different identifications, delusions, illusions, and hallucinations result. Their ‘ideas’ are evaluated as things or experience, and affectively objectified in different degrees, which results in the above mis-evaluating manifestations. These semantic disturbances and tensions make the ‘mentally’ ill believe irresistibly in the ‘truth’ of their ‘premises’ and their inductions and deductions, which they follow blindly. In them, as in the rest of us, some internal affective pressure comes first, but because in humans the effect of higher nerve centres cannot be entirely abolished, this affective pressure is rationalized somehow into some sort of ‘premises’. This organism-as-a-whole process is entirely general and applies to all of us in all our activities, but is most clearly seen in the ordered details in the work of creative scientists and ‘geniuses’, and in the more severe cases of ‘mental’ illness. To the ‘mentally’ ill these ‘premises’ have the value of ‘the’ and not ‘a’ premise. *They act upon them*, and so cannot adjust themselves to a world different from their fancies. They would seldom survive at all if left alone by themselves, particularly in a complex ‘civilization’.

Mathematicians, also, have structural premises, often called postulates, but they *never* evaluate them to be ‘true’; wherefore their premises *cannot* be ‘false’. They have no claims, and claims are always affective. Like the ‘insane’, they follow up these premises blindly, but, being generally conscious of abstracting in the field of their profession, they are not usually subject to semantic disturbances *in this field* and do not live out their theories in life, the theories thus remaining affectively hypothetical. If a mathematician were to believe, with strong affective evaluation, that his premises are ‘true’, these premises then would become mostly false, or meaningless, or, in general, inappropriate. If he lived through them, the given individual would then be ‘mentally’ ill, not because of his premises, but because of the semantic disturbance, which would involve erroneous evaluation, identifications, confusion of orders of abstractions in his affective *attitude toward his premises.* This subtle organism-as-a-whole mechanism, in which all affective pressure

can be rationalized, and all rationalization can produce affective manifestations, not only makes the present *non-el* analysis possible and legitimate, but also offers some explanation of those remarkable cases of ‘mental’ illness in a number of mathematical geniuses. Under such organism-as-a-whole structural conditions, a *general* consciousness of abstracting not restricted to a special field is the only possible safeguard against the semantic disturbances which lead to an unbalanced ‘mental’ condition.

As we have seen, the difference between ‘sanity’ and ‘insanity’ is subtle. The reader must be reminded that it takes a good ‘mind’ to be ‘insane’. Morons, imbeciles, and idiots are ‘mentally’ deficient, but could not be ‘insane’.

The so-called ‘sane’ also have structural premises; we all have some standards of evaluation. These are also usually false, or, in general, inappropriate, being mostly due to our savage inheritance. But the saner we are, the less we abide by them. Therefore, in a world quite different structurally from our fancies, we are often able to adjust ourselves for all practical purposes, often avoiding major disasters for a number of years.

For instance, the believers in extraordinary blisses in the ‘other life’ or the ‘other world’ should welcome death. Why be so unhappy here, when, according to their doctrines, there is such an ideally happy future after death ? Why make use of medicine and doctors, when a deadly illness should open the door to everlasting bliss ! In conflict with such a creed, he lives as long as he can, often most unhappily, and is generally willing to spend fortunes on doctors and medicines to delay the bliss ! The genuine and very serious danger to all of us of such creeds is that when the *s.r* of an individual are trained in this way he finally does become indifferent, or apathetic toward actualities in *this world*, so that cunning, and often pathological, individuals are thus given an opportunity of directing human affairs toward their personal ends.

Naturally, with the increase of the complexities of conditions, the dangers also increase in a geometrical ratio, because when *m.o* realities become too unbearable, the masses cease to be influenced by these semantic illusions, and they break all barriers, only to fall again under the influence of new leaders very often equally irresponsible and ignorant.

Unfortunately, the failure to understand these semantic issues, based on animalistic lack of foresight, results invariably in a great deal of unnecessary suffering. There is little doubt that without these delusions and illusions we should look after the conditions of our actual lives more closely, and many of our pressing needs would be adjusted.

The difficulties which we have are mostly man-made, and so only mankind can remedy them, and any attempts to escape from *m.o* reality only aggravate the situation.

Lack of space does-not allow me to dwell here on many other aspects of mathematics which are of neurological structural importance, except to mention the theory of statistics and probability. All human knowledge is neurologically due to a process of abstracting in different orders, giving us the only structural knowledge of processes, which, in 1933, must always be considered on three levels, the macroscopic, the microscopic, and the sub-microscopic.

Because the nervous system is an abstracting, integrating mechanism, all human psycho-neurological reactions and, particularly, psycho-logical, to be similar in structure, *must* be based on the mathematical theories of statistics and *probability.* On the objective level, we deal with absolute individuals, and so all statements, or higher order abstractions, can only be probable. Historically, mathematicians have elaborated not only both theories, but Boole, in his *Laws of Thought,* extended the mathematical approach to 'logic' in connection with the theory of probability. Finally, the difficulties of the law of excluded third have been solved by Lukasiewicz and Tarski[10] in their 'many-valued logic', which, when *N* increases indefinitely, merges with the mathematical theory of probability, a result reached independently by a different type of analysis in the present system. Any possible future scientific $\overline{A}$, *non-el* 'logic', which I call general semantics, must be built on this structurally more correct foundation. It should be noticed that the notions of probability are very flexible, and entirely cover our structural needs, the field of degrees of probability ranging from impossibility to certainty. This new semantics involves entirely new affective attitudes, and underlies new and better balanced *s.r.*

Under such conditions, the restricted 'uncertainty principle' of Heisenberg becomes a structural, most revolutionary, and creative *general principle,* transferring the laws of two-valued 'cause and effect' from the realm of gambling on words by 'philosophers' to the scrutiny of scientists, and establishing ∞-valued 'determinism' on a neuro-mathematical base of 'the greatest probability'. Methodologically and psycho-logically, this requires *full consciousness of abstracting,* achieved, as yet, by extremely few of us, even among physicists and mathematicians. Then the 'law' of two-valued 'cause and effect', instead of depending on the *el* and objectified older interpretations, will be based on the mathematical, and much more reliable, ∞-valued principle of greatest proba-

bility. This will eliminate to a large extent semantic disturbances, and so the problems of sanity will be greatly helped towards solution.

To those who are accustomed to the disclaimers made by many mathematicians of human values in their work, such an analysis as I have given in the present chapter must seem unexpected. But, upon reflection, we may see that, after all, it is only a natural evaluation. Language is a unique, and, therefore, most important, human characteristic. Ought we to wonder that these linguists of exact sciences, whom we call mathematicians, should have produced unknowingly and unwittingly great human values, fundamentally affecting the *s.r* ? They could not help it. Once they worked out their own problems properly—and no one doubts that they did it well—the results were bound to have broad human significance. Their activities were kept on the proper levels and so were naturally a help toward sanity. In Part VII, I shall discuss another mathematical discovery, known as the 'theory of mathematical types', of Russell, which, when generalized, becomes a *physiological* theory of enormous semantic importance and of fundamental and constant human application.

In spite of popular belief, mathematics is the simplest language in existence. Our daily language is so very complex in its structure that for many thousand years it evaded analysis. Probably, the writer, without the study of mathematics, would not have been able to discover the ultimately extremely simple yet workable principles outlined in the present work.