

## CHAPTER XV

### THE 'INFINITESIMAL' AND 'CAUSE AND EFFECT'

But we are not likely to find science returning to the crude form of causality believed in by Fijians and philosophers, of which the type is "lightning causes thunder." (457)

BERTRAND RUSSELL

The notion of causality has been greatly modified by the substitution of space-time for space and time. . . . Thus geometry and causation becomes inextricably intertwined. (457)

BERTRAND RUSSELL

In classical mechanics, and no less in the special theory of relativity there is an inherent epistemological defect which was, perhaps for the first time, clearly pointed out by Ernst Mach.... No answer can be admitted as epistemologically satisfactory, unless the reason given is an *observable fact of experience*. The law of causality has not the significance of a statement as to the world of experience, except when *observable facts* ultimately appear as causes and effects. (155)

A. EINSTEIN

The chain of cause and effect could be quantitatively verified only if the whole universe were considered as a single system—but then physics has vanished and only a mathematical scheme remains. The partition of the world into observing and observed system prevents a sharp formulation of the law of cause and effect. (215)W.  
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Of late, another perplexing semantic problem concerning 'causality' or 'non-causality' has arisen in connection with the newer quantum mechanics. It is possible to examine this question by different methods. The simpler one is connected with vague feelings of 'infinity' and its supposed opposite, the 'infinitesimal'; the more fundamental method is based on the orders of abstractions leading toward the  $\infty$ -valued semantics of probability.

Because of man's natural tendency to speak in terms of 'infinity', and his further marked tendency of having opposites, such as 'yes', 'no', 'right', 'left', 'positive', 'negative', 'love', 'hate', 'honesty', 'dishonesty', quite naturally the notion of 'infinity' carried with it the tendency to invent the 'infinitesimal'. Even mathematicians have had great semantic difficulties in breaking away from this habit. Analysis persistently reveals that structurally no matter how far we go in dividing something, let us say an inch, whatever is left may be extremely small, but yet it is a perfectly good finite quantity. Thus, structural difficulties were encountered with the postulated 'infinitesimal'. The name implies that they are not finite, yet analysis shows only finites. Mathematicians supposed that an 'infinitesimal' was necessary for mathematics, and so they were reluctant to abandon it.

The 'infinitesimal', like so many other baffling suppositions, was invented by

the Greeks, who regarded a circle as differing 'infinitesimally'

from a polygon with a very large number of very small equal sides. With the invention of the differential and integral calculus, 'infinitesimal calculus', as it was called, the importance of the 'infinitesimal' increased, and even mathematicians used it as a fundamental notion. Finally, Weierstrass succeeded in showing the meaningless character of the 'infinitesimal', and also that the 'infinitesimal' was not structurally necessary for the calculus. Up to that date, the problem was baffling; we knew that the calculus required 'continuity', which, in turn, seemed to require 'the infinitely little', and yet no one could tell what this 'infinitely little' might represent. It was quite obviously not zero, because a sufficient number of them was able to make up a finite whole; and we knew no fraction which was not zero and yet not finite. The discovery by Weierstrass that the calculus does not require the 'infinitesimal', and that all deductions could be made without it, abolished a very serious structural, verbal, metaphysical, and semantic bugaboo. Common sense, of course, is much simpler, although unreliable in such matters, and was satisfied also.

The elimination of the 'infinitesimal' is a great semantic step forward, and helps to clarify structurally some deeply rooted, vague, fallacious notions, which are overloaded with affective components and are extremely vicious in their effects.

If there is no 'infinitesimal', there is no 'next moment'; for the interval between any two moments must be finite, and so there are always other moments in the interval between them. Also, two moments cannot be consecutive, for between any two there are always other moments, no matter how far we go; similarly, the 'present' becomes a very vague notion.

For our purpose, the most fundamental semantic application of what has been said above is in the vast field embraced by the old structural notions of 'cause' and 'effect'. These terms are of great antiquity, of a distinctly pre-scientific one-, two-valued semantic epoch. They originated in the rough experience of our race, and are firmly rooted in the habits of 'thought' and the structure of our old two-valued 'logic' and language, and because of that are even now unduly baffling. These terms, in the *two-valued sense*, were and are the structural assumptions of our 'private' and 'official' 'philosophies'. The unenlightened use of these terms has done much to prevent the formulation of a science of man and to build up vicious anti-scientific metaphysics of various sorts involving pathological *s.r.* With the new quantum mechanics, a better understanding of these notions, based on the  $\infty$ -valued semantics of probability, becomes a paramount issue for all science. In daily life, the

indiscriminate use of two-valued 'cause' and 'effect' leads structurally to a great deal of absolutism, dogmatism, and other harmful semantic disturbances, which I call confusion of orders of abstraction.

We usually follow the 'philosophers' and ascribe—or, rather feel, as conscious ascribing would not stand criticism—some mysterious structural continuity, some mysterious overlapping of 'cause' and 'effect'. We 'feel', and try to 'think', about 'cause and effect' as *contiguous* in 'time'. But 'contiguous in time' involves the impossible 'infinitesimal' of some unit of 'time'. But, since we have seen that there is no such thing, we must accept that the interval between 'cause' and 'effect' is finite. This structural fact changes the whole situation. If the interval between 'cause' and 'effect' is finite, then always something might happen between, no matter how small the interval may be. The 'same cause' would not produce the 'same effect'. The expected result would not follow. This means only that in this world, to be sure of some expected effect, requires that there must be nothing in the environment which can interfere with the process of passing from the conditions labelled 'cause' to the conditions labelled 'effect'. In this world, with the structure which it has, we can never suppose that a 'cause', as we know it, is *alone* sufficient to produce the supposed 'effect'. When we consider the ever-changing environment, the number of possibilities increases enormously. If it were possible to take into account the *whole* of the environment, the *probability* that some event would be repeated, in all details, thus exhibiting the assumed two-valued relation of 'cause' and 'effect', which we took for granted in the old days, would practically be nil. The principle of non-elementalism, as we see, requires an  $\infty$ -valued semantics of probability.

The reader should not take what is said here as a denial that in this external world some regularities of sequence occur; but the above analysis, which is mainly due to Russell,<sup>1</sup> shows clearly that the verbal principle of 'same cause, same effect' is structurally untenable. We can never manage to observe the 'same cause' in detail. As soon as the antecedents have been sufficiently ascertained, so as to calculate the consequences with some plausible accuracy as to details, the relations of these antecedents have become so complex that there is very little *probability* that they will ever occur again.

The clearing up of the problems of 'cause' and 'effect' is of serious importance, because powerful semantic reactions are connected with it. To begin with, we must differentiate between the terms 'cause' and 'effect', which, linked together, imply a two-term relation nowhere to be found in this world, and thus represent a language and a two-valued

'logic' of a structure not similar to the structure of the world, and the *general  $\infty$ -valued notion of causality*. This last notion is the psycho-logical foundation of all explanations leading toward  $\infty$ -valued determinism, and is an exclusive test for structure; and so of extreme semantic importance.

Besides the analysis from the point of view of the impossible 'infinitesimal', the term 'cause-effect' represents a two-term relation, and, as such, is a primitive generalisation *never* to be found in this world, as all events are *serially* related in a most complex way, independent of our way of speaking about them. If we expand our two-term relation 'cause-effect' into a *series*, we pass from the inferential level to the *descriptive* level, and so can apply a behaviouristic, functional, actional language of order. In such series, we could only use the language of 'cause' and 'effect' if we could select neighbouring factors, a selection which is often impossible. Also, if we pass from macroscopic to microscopic or sub-microscopic levels, we could use such language, but then the terms would have different meanings, supplied by the theory of probability.

The semantic side of this problem is of importance, because, in the old *el* way, it was neglected. General speculations about such *m.o* terms as 'cause' and 'effect' are useless. Such statements are not propositions, but involve variable meanings and, therefore, generate propositional functions which are neither true nor false. Our expanding of the too simple, two-term relation 'cause-effect' into a complex series is closer to the structure of this world, as far as we know it.

The understanding and habitual application of what has just been said would not only save us from silly dogmatizing and inappropriate *s.r*, but would teach us not to disregard any regularity, and to investigate any relation which might appear. Then, in a *specific case*, we could again use the restricted principle of causality, based on probability and averages. The old absolute and objectified semantic attitude toward 'cause-effect' was and often is a serious hindrance in observing impartially the sequence of events (order) and relations. Preconceived notions and old *s.r* played havoc, for it is well known that we usually find what we want to find. If we approach a problem with definite unconscious '*emotional*' wants, and cannot satisfy these *s.r*, we become bewildered, down-hearted, and perhaps utter some such nonsense as the 'finite mind', or the like. Under such semantic pressure, our power of observation and analysis is reduced by a kind of 'emotional stupor'. Such an occurrence is harmful in science and in life. 'Human knowledge' depends on human ingenuity, power of observation, power of abstraction, . It is an activity

of the human nervous system inside of our skin and can never be the events themselves.

We see that the old two-valued verbal structure of 'cause' and 'effect' is not similar to the structure of the world, but a rash limiting generalization from probability. Since these expressions belong to the class of statistical averages and depend on the scale of the events and intervals dealt with, we must not expect that such terms as two-valued 'causality', which is a term of statistical macroscopic averages, will apply in that sense to small-scale events when the intervals are much smaller and when entirely different conditions and 'causes' prevail. Today we have structural evidence that even 'space' and 'time' represent statistical averages and do not apply to the smallest scale events. It is natural that 'cause' and 'effect' should join their company. The above involves epistemologically the passing from the  $A$  two-valued system to a  $\bar{A}$   $\infty$ -valued system. Psychophysically, it involves new *s.r.*

In mathematics, the old religious attitude toward the 'infinitesimal' is rapidly vanishing. Many mathematicians deliberately, and justly, avoid the use of the word. A term like 'indefinitely small' or 'indefinitesimal' is a better descriptive term, truer in its implications. We even see scientists like Eddington, who had the pluck—it is still pluck, unfortunately—to treat enormous stellar distances as 'infinitesimals of second order'. ('Infinitesimal' is used here in a mathematical sense of indefinitesimal.)

It has been already mentioned that most of the important discoveries of mathematics were due to a special semantic attitude on the part of those who made them. This attitude was an unconscious or conscious treatment of mathematics as a form of human behaviour. We see an example in the work of Weierstrass and his analysis of the 'infinitesimal'. He did not take the 'infinitesimal' as some objectified metaphysical structure and remain content; he analysed the *genetic process* by which the 'infinitesimal' was *made* by Smith and Brown, and so treated mathematics structurally as a form of human behaviour. Any deepening of the foundations, or clarification of fundamental notions, or investigation of underlying assumptions, *must*, by necessity, have this characteristic. The man who does it must take into account how the given process was produced—analyse its structure, and so start with the ways and methods of production. In other words, he must treat the given problem as a form of human behaviour. The fact that this simple, and quite obvious method has been formulated and structurally explained as *desirable* is helpful. It shows the *method* and structure of the path by which advances can be reached. We can *train* the semantic reactions of students to it and make progress inevitable; but now, instead, it takes a genius to break, by him-

self, through the old semantic habits which have been produced by the lack of scientific psycho-logics and training.

The term 'correct symbolism' has already been used. In this world of structurally absolute individuals, the minimum of structurally desirable correct symbolism must provide for the possibility of labelling these absolute individuals by separate names. For scientific purposes, we must use terms built on the pattern of mathematical symbolism; i.e., according to the *extensional* methods. We must adopt a behaviouristic attitude and habits in our term-making. As we proceed, we must emphasize *order*, considering what comes first and what next. This is semantically important, for the usual procedure is entirely different: first, we have our structurally 'preconceived' doctrines and languages; next, we observe the structure of the world; and *then* we try to force the observed facts into the linguistic structural patterns. But, in the new way, we *start* with silent observations, and search empirically for structure; next, we invent verbal structures similar to them; and, finally, we see what can be said about the situation, and so test the language. Experience shows that the old habits of labels first, objects next, instead of the structurally natural order of *objects first, labels next*, is semantically pernicious and harmful. In Part VII, it is shown that the semantic structural reversal of the unnatural reversed order is crucial for sanity.

From the days of the Greeks an acute difficulty has made itself felt; namely, how to reconcile the world of physics with the world of mathematics. For mathematics, we need 'extensionless' points; for physics, we need finite-sized elements. Whitehead and Russell have suggested different structures by which this may be accomplished. It seems possible to demand that none of the material dealt with shall be smaller than an assigned finite size. That this condition can be reconciled with mathematical continuity seems to be novel. Whether this device is valid or not, it is yet too early to decide. This problem of reconciliation will become important further on when we come to speak of events as made up from point-events.<sup>2</sup>