PART IV

STRUCTURAL FACTORS IN NON-ARISTOTELIAN LANGUAGES

Without objects conceived as unique individuals, we can have *no Classes.* Without classes we can, as we have seen, define *no Relations*, without relations we can have *no Order. But to be reasonable is to conceive of order-systems, real or ideal. Therefore, we have an absolute logical need to conceive of individual objects as the elements of our ideal order systems.* This postulate is the condition of defining clearly any theoretical conception whatever. The further metaphysical aspects of the concept of an individual we may here ignore. *To conceive of individual objects is a necessary presupposition of all orderly activity.* (449)

JOSIAH ROYCE

The connections shown by these particular examples hold in general: given a transformation, you have a function and a relation; given a function, you have a relation and a transformation; given a relation, you have a transformation and a function: *one* thing—*three* aspects; and the fact is exceedingly interesting and weighty. (264) CASSIUS J. KEYSER

It can, you see, be said, with the same approximation to truth, that the whole of science, including mathematics, consists in the study of transformations or in the study of relations. (264) CASSIUS J. KEYSER

Science is never merely knowledge; it is orderly knowledge. (449)

JOSIAH ROYCE

Philosophers have, as a rule, failed to notice more than two types of sentence, exemplified by the two statements “this is yellow” and “buttercups are yellow.” They mistakenly suppose that these two were one and the same type, and also that all propositions were of this type. The former error was exposed by Frege and Peano; the latter was found to make the explanation of order impossible. Consequently the traditional view that all propositions ascribe a predicate to a subject collapsed, and with it the metaphysical systems which were based upon it, consciously or unconsciously. (22) BERTRAND RUSSELL

Interesting analyses by Van Woerkom have shown a general incapacity in aphasics for grasping relations, realizing ordered syntheses, etc.; all of them are operations which are based, in the normal individual, on the use of verbal symbolization. When confronted by groups of figures or of geometrical forms, the aphasic, even though he may perceive them correctly, is unable to analyse or to order the elements, to grasp their succession . . . (411) HENRI PIÉRON

CHAPTER XI

ON FUNCTION

The whole science of mathematics rests upon the notion of function, that is to say, of dependence between two or more magnitudes, whose study constitutes the principal object of analysis. C. E. PICARD

Every one is familiar with the *ordinary* notion of a function—with the notion, that is, of the lawful dependence of one or more variable things upon other variable things, as the area of a rectangle upon the lengths of its sides, as the distance traveled upon the rate of going, as the volume of a gas upon temperature and pressure, as the prosperity of a throat specialist upon the moisture of the climate, as the attraction of material particles upon their distance asunder, as prohibitionary zeal upon intellectual distinction and moral elevation, as rate of chemical change upon the amount or the mass of the substance involved, as the turbulence of labor upon the lust of capital, and so on and on without end. (264)

CASSIUS J. KEYSER

The infinite which it superficially gets rid of is concealed in the notion of "any," which is but one of the protean disguises of mathematical generality. (22)

E. T. BELL

The famous mathematician, Heaviside, mentions the definition of quaternions given by an American schoolgirl. She defined quaternions as 'an ancient religious ceremony'. Unfortunately, the attitude of many mathematicians justified such a definition. The present work departs widely from this religious attitude and treats mathematics simply as a most important and unique form of human behaviour. There is nothing sacred about any single verbal formulation, and even those that now seem most fundamental should be held subject to structural revision if need should arise. The few mathematicians who have produced epoch-making innovations in mathematical method had this behaviouristic attitude *unconsciously,* as will be shown later. The majority of mathematicians take mathematics as a clear-cut entity, 'by itself'. This is due, first, to a confusion of orders of abstractions and to identification, as will be explained later; and, second, to its seeming simplicity. In reality, such an attitude introduces quite unexpected complications, leading to mathematical revolutions, which are always bewildering. The mathematical revolutions occur only because of this *over-simplified,* and thus fallacious, attitude of the mathematicians toward their work. Had all mathematicians the semantic freedom of those who make the mathematical 'revolutions', there would be *no* mathematical 'revolutions', but an extremely swift and constructive progress. To re-educate the *s.r* of such mathematicians, the problem of the psycho-logics of mathematics must receive more attention. This means that some mathematicians must become psycho-logicians also, or that psycho-logicians must study mathematics.

For, let us take a formula which exemplifies mathematics at its best; namely, one and one make two (1 + 1=2). We see clearly that this human product involves a threefold relation: between the man who made it,

$$\text{(A)} \qquad \begin{tikzcd} & 1+1=2 \arrow[dl] & \\ \text{Brown} \arrow[rr, leftrightarrow] & & \text{Smith} \arrow[ul] \end{tikzcd}$$

let us say, Smith, and the black-on-white marks (A), between these marks and Brown, and between Brown and Smith. This last relationship is the only *important* one. The marks (A) are only auxiliary and are *meaningless by 'themselves'*. They would never occur if there were no Smiths to make them, and would be of no value if there were no understanding Browns to use and to appreciate them. It is true that when we take into account this threefold relation the analysis becomes more difficult and must involve a revision of the foundations of mathematics. Although it is impossible to attempt in this book a deeper analysis of these problems in a general way, yet this behaviouristic attitude follows the rejection of the 'is' of identity, and is applied all through this work.

The notion of 'function' has played a very great role in the development of modern science, and is structurally and semantically fundamental. This notion was apparently first introduced into mathematical literature by Descartes. Leibnitz introduced the term. The notion of a 'function' is based on that of a *variable*. In mathematics, a variable is used as an ∞-valued symbol that can represent *any one* of a series of numerical elements.

It is useful to enlarge the mathematical meaning of a variable to include any ∞-valued symbol of which the value is not determined. The various determinations which may be assigned to the variable we call the *value* of the variable. It is important to realize that a mathematical variable does not vary or change in itself, but can take any value within its range. If a particular value is selected for a variable, then this value, and, therefore, the variable, becomes fixed—a one-valued constant. In the use of these terms, we should take into account the behaviour of the mathematizer. His '*x*' is like a container, into which he may pour any or many liquids; but once the selection has been made, the content of the container is one or a constant. So 'change' is not inherent in a variable; it is due only to the volition of the mathematizer, who can change one value for another. Thus, the value changes by quanta, in definite lots, according to the pleasure of the operator. This quantum character of the variable has serious structural and semantic consequences, which will become clearer further on. It allows us, without stretching our definitions, to apply the new vocabulary to any problem whatsoever. It is in structural accord with the trend of the quantum theory, and, therefore, with the *structure* of this world, as we know it at present.

The notion of a variable originated in mathematics, and, in the beginning, dealt only with numbers. Now numbers, when given, represent, structurally, a manifold or aggregate which is *not* supposed to change. So, when we consider a variable, we should ‘think’ not of a changing. entity, but of *any* element we choose out of our perfectly constant collection (when given). Let me repeat that the notion of *change* enters in, only in connection with the volition and the *s.r* of the one who operates these unchanging entities. The notion of a variable is taken always in an extensional ∞-valued sense, to be explained later, as it always implies structurally a collection of many individuals, out of which collection a selection of one can be made. The notion of a variable is general and, in principle, ∞-valued; a constant is a special one-valued case of a variable in which the collection contains a single element, making alternative selection impossible.

Variables are usually symbolized by the end letters of the alphabet, x, y, z, . The supply is increased as desired by the use of indices; for instance, x', y', z'. ; x'', y'', z''. ; or x_1, y_1, z_1. ; x_2, y_2, z_2, . This gives a flexible means of denoting numerous individuals, and so manufacturing them indefinitely, as the extensional method of mathematics requires. Another method, introduced not long ago, has proven useful in dealing with a definite selection of variables in a simplified manner. One letter or one equation can be used instead of many. The variable sign x is modified by another letter which may have different values, in a given range; for instance, x_i, x_k, . The modifying letter i or k can take the serial values; let us say i or k=1, 2, 3, . Since the one symbol x_k stands for the array of many *different* variables x_1, x_2, x_3. , statements can be greatly simplified, and yet preserve structurally the *extensional* individuality.

It is important that the non-mathematical reader should become acquainted with the above methods and notations, as they involve a profound and far-reaching structural and *psycho-logical* attitude, useful to *everybody*, involving most fundamental *s.r.*

The *extensional* method means dealing structurally with many *definite individuals;* as, for instance, with 1, 2, 3. , a series in which each individual has a special and *unique* name or symbol. This extensional method is structurally the *only* one by which we may expect to acquire $\overline{A}$ ∞-valued *s.r*. In a strict sense, the problems in life and the sciences do not differ structurally from this mathematical problem. In life and science, one deals with many, actual, unique individuals, and all *speaking* is using abstractions of a very high order (abstractions from abstraction from abstraction, .). So, whenever we speak, the individual is never completely covered, and some characteristics are left out.

A rough definition of a function is simple: *y* is said to be a function of *x*, if, when *x* is given, *y* is determined. Let us start with a simple mathematical illustration: $y=x+3$. If we select the value 1 for *x* our $y=1+3=4$. If we select $x=2$, then $y=2+3=5$, . Let us take a more complicated example; for instance: $y=x^2-x+2$. We see that for $x=1$, $y=1-1+2=2$;for $x=2$, $y=4-2+2=4$; for $x=3$, $y=9-3+2=8$, .

In general, *y* is determined when we fulfill all the indicated *operations* upon the variable *x*, and so get the final results of these operations. In symbols, $y = f(x)$, which is read, *y* equals function of *x*, or *y* equals *f* of *x*.

In our example, we may call *x* the independent variable, meaning that it is the one to which we may assign any value at our pleasure, if not limited by the conditions of our problem, and *y* would then be the dependent variable, which means that its value is no longer dependent on our pleasure, but is determined by the selection of the value of *x*. The terms dependent and independent variables are not absolute, for the dependence is mutual, and we could select either variable as the independent one, according to our wishes.

The notion of a 'function' has been generalized by Bertrand Russell to the very important notion of a 'propositional function'.[1] For my purpose, a rough definition will be sufficient. By a propositional function, I mean an ∞-*valued* statement, containing one or more variables, such that when single values are assigned to these variables the expression becomes, in principle, a *one-valued* proposition. The ∞-valued character of propositional functions seems essential, because we may have a one-valued descriptive function with variables, or a one-valued expression formulating a semantic relational law expressed in variable terms. , yet these would be propositions. Thus, the ∞-*valued* statement, '*x* is black', would exemplify a propositional function; but the one-valued relation 'if *x* is more than *y*, and *y* is more than *z*, then *x* is more than *z*' exemplifies a proposition. This extended *m.o* notion of a propositional function becomes of crucial importance in a $\overline{A}$-system, because most of our speaking is conducted in $\overline{A}$ ∞-valued languages to which we mostly and delusionally ascribe single values, entirely preventing proper evaluation.

An important characteristic of a propositional function, for instance, '*x* is black', is that such a statement is neither true nor false, but ambiguous. It is useless to discuss the truth or falsehood of propositional functions, since the terms true or false cannot be applied to them. But if a definite, single value is assigned to the variable *x*, then the propositional function becomes a proposition which may be true or false. For instance, if we assign to *x* the value 'coal', and say 'coal is black', the ∞-valued propositional function has become a one-valued true proposition. If we should assign to *x* the value 'milk', and say 'milk is black', this also would make a proposition, but, in this case, false. If we should assign to *x* the value 'blah-blah', and say 'blah-blah is black', such a statement may be considered as meaningless, since it contains sounds which have *no* meaning; or we *may* say, 'the statement blah-blah is *not* black but meaningless', and, therefore, the proposition 'blah-blah is black', is *not* meaningless but false.

We should notice—a fact disregarded in the *Principia Mathematica*—that there is no hard and fast rule by which we can distinguish between meaningless and false statements in general, but that such discrimination depends on many factors in each specific case. A propositional function, '*x* is black', cannot be its own argument: for instance, if we substitute the whole propositional function, '*x* is black', for the variable *x* in the original propositional function, and then consider the expression, '*x* is black is black', which Whitehead and Russell classify as *meaningless,* this expression is not necessarily meaningless, but *may* be considered *false.* For, the statement, '*x* is black', is defined as a *propositional function,* and, therefore, the statement, 'x is black, is black', may be considered *false.*

The problems of 'meaning' and 'meaningless' are of great semantic importance in daily life, but, as yet, little has been done, and little research made, to establish or discover valid criteria. To prove a given statement false is often laborious, and sometimes impossible to do so, because of the undeveloped state of knowledge in that field. But with meaningless verbal forms, when their meaninglessness is exposed in a given case, the non-sense is exploded for good.

From this point of view, it is desirable to investigate more fully the mechanism of our symbolism, so as to be able to distinguish between statements which are false and verbal forms which have no meanings. The reader should recall what was said about the term 'unicorn', used as a symbol in heraldry and, eventually, in 'psychology', since it stands for a human *fancy*, but, in zoology, it becomes a noise and not a symbol, since it does not stand for any actual animal whatsoever.

A very curious semantic characteristic is shared in common by a propositional function and a statement containing meaningless noises; namely, that neither of them can be true or false. In the old *A* way all sounds man made, which could be written down and looked like words, were considered words; and so every 'question' was expected to have an answer. When spell-marks (noises which can be spelled) were put together in a specified way, each combination was supposed to say something, and this statement was supposed to be true or false. We see clearly that this view is not correct, that, in addition to words, we make noises (spell-marks) which may have the appearance of being words, but should *not* be considered as words, as they say nothing in a given context. Propositional functions, also, cannot be classified under the simple two opposites of true and false.

The above facts have immense semantic importance, as they are directly connected with the possibility of human agreement and adjustment. For upon statements which are neither true nor false we can always disagree, if we insist in applying criteria which have no application in such cases.

In *human* life the semantic problems of 'meaninglessness' are fundamental for sanity, because the evaluation of noises, which do not constitute symbols in a given context as symbols in that context, must, of necessity, involve delusions or other morbid manifestations.

The solution of this problem is simple. Any noises or signs, when used semantically as symbols, *always* represent *some symbolism,* but we must find out to what field the given symbolism applies. We find only three-possible fields. If we apply a symbol belonging to one field to another field, it has very often no meaning in this latter. In the following considerations, the theory of errors is disregarded.

A symbol may stand for: (1) Events outside our skin, or inside our skin in the fields belonging to physics, chemistry, physiology, . (2) Psycho-logical events inside our skin, or, in other words, for *s.r* which may be considered 'sane', covering a field belonging to psycho-logics. (3) Semantic disturbances covering a pathological field belonging to psychiatry.

As the above divisions, together with their interconnections, cover the field of human symbolism, which, in 1933, have become, or are rapidly becoming, *experimental* sciences, it appears obvious that older 'metaphysics' of every description become illegitimate, affording only a very fertile field for study in psychiatry.

Because of *structural* and the above *symbolic* considerations based on $\bar{A}$ negative, non-identity premises, these conclusions appear as *final;* and, perhaps, for the first time bring to a focus the age-long problem of the subject-matter, character, value, and, in general, the status of the older 'metaphysics' in human economy. From the *non-el,* structural, and semantic point of view, the problems with which the older 'metaphysics' and 'philosophy' dealt, should be divided into two quite definite groups. One would include 'epistemology', or the theory of knowledge, which would ultimately merge with scientific and *non-el* psycho-logics, based on general semantics, structure, relations, multi-dimensional order, and the quantum mechanics of a given date; and the rest would represent semantic disturbances, to be studied by a generalized up-to-date psychiatry.

Obviously, considerations of structure, symbolism, sanity. , involve the solutions of such weighty problems as those of 'fact', 'reality', 'true', 'false'. , which are completely solved only by the consciousness of abstracting, the multiordinality of terms. ,—in general, a $\bar{A}$-system.

Let me repeat the rough definition of a propositional function—as an ∞-valued statement containing variables and characterized by the fact that it is ambiguous, neither true nor false.

How about the terms we deal with in life ? Are they all used as one-valued terms for constants of some sort, or do we have terms which are inherently ∞-valued or variable ? How about terms like 'mankind', 'science', 'mathematics', 'man', 'education', 'ethics', 'politics', 'religion', 'sanity', 'insanity', 'iron', 'wood', 'apple', 'object', and a host of other terms ? Are they labels for one-valued constants or labels for ∞-valued stages of processes. Fortunately, here we have no doubt.

We see that a large majority of the terms we use are names for ∞-valued stages of processes with a *changing content.* When such terms are used, they generally carry different or many contents. The terms represent ∞-valued variables, and so the statements represent ∞-valued propositional functions, not one-valued propositions, and, therefore, in (principle, are neither true nor false, but ambiguous.

Obviously, before such propositional functions can become propositions, and be true or false, single values must be assigned to the variables by some method. Here we must select, at least, the use of co-ordinates. In the above cases, the 'time' co-ordinate is sufficient. Obviously, 'science 1933' is quite different from 'science 1800' or 'science 300 B.C'.

The objection may be made that it would be difficult to establish means by which the use of co-ordinates could be made workable. It seems that this might involve us in complex difficulties. But, no matter how simple or how complex the means we devise, the details are immaterial and, therefore, we can accept the roughest and simplest; let us say, the year, and usually no spatial co-ordinates. The invaluable semantic effect of such an innovation is *structural,* one-, versus ∞-valued, *psycho-logical* and methodological, and affects deeply our *s.r*.

From time immemorial, some men were supposed to deal in one-valued ‘eternal verities’. We called such men ‘philosophers’ or ‘metaphysicians’. But they seldom realized that all their ‘eternal verities’ consisted only of *words,* and words which, for the most part, belonged to a primitive language, reflecting in its structure the assumed structure of the world of remote antiquity. Besides, they did not realize that these ‘eternal verities’ last only so long as the human nervous system is not altered. Under the influence of these ‘philosophers’, two-valued ‘logic’, and confusion of orders of abstractions, nearly all of us contracted a firmly rooted predilection for ‘general’ statements—’universals’, as they were called—which, in most cases, inherently involved the semantic one-valued conviction of validity for all ‘time’ to come.

If we use our statements with a date, let us say ‘science 1933’,such statements have a profoundly modified structural and psycho-logical character, different from the old general legislative semantic mood. A statement concerning ‘science 1933’, whether correct or not, has no element of semantic conviction concerning 1934.

We see, further, that a statement about ‘science 1933’ might be quite a definite statement, and that if the person is properly informed, it probably would be true. Here we come in contact with the structure of one of those human semantic impasses which we have pointed out. We humans, through old habits, and because of the inherent structure of human knowledge, have a tendency to make static, definite, and, in a way, absolutistic one-valued statements. But when we fight absolutism, we quite often establish, instead, some other dogma equally silly and harmful. For instance, an active atheist is psycho-logically as unsound as a rabid theist.

A similar remark applies to practically all these opposites we are constantly establishing or fighting for or against. The present structure of human knowledge is such, as will be shown later, that we tend to make definite statements, static and one-valued in character, which, when we take into account the present pre-, and *A* one-, two-, three-valued affective components, inevitably become absolutistic and dogmatic and extremely harmful.

It is a genuine and fundamental semantic impasse. These static statements are very harmful, and yet they cannot be abolished, for the present. There are even weighty reasons why, without the formulation and application of ∞-valued semantics, it is not possible (1933) to abolish them. What can be done under such structural circumstances ? Give up hope, or endeavour to invent methods which cover the discrepancy in a satisfactory (1933) way ? The analysis of the psycho-logics of the mathematical propositional function and $\bar{A}$ semantics gives us a most satisfactory structural solution, necessitating, among others, a four-dimensional theory of propositions.

We see (1933) that we can make definite and *static* statements, and yet make them semantically *harmless*. Here we have an example of abolishing one of the old *A* tacitly-assumed ‘infinities’. The old ‘general’ statements were supposed to be true for ‘all time’; in quantitative language it would mean for ‘infinite numbers of years’. When we use the date, we reject the fanciful tacit *A* ‘infinity’ of years of validity, and limit the validity of our statement by the date we affix to it. Any reader who becomes accustomed to the use of this structural device will see what a tremendous semantic difference it makes psycho-logically.

But the above does not exhaust the question structurally. We have seen that when we speak about ∞-valued processes, and stages of processes, we use variables in our statements, and so our statements are not propositions but propositional functions which are not true or false, but are ambiguous. But, by assigning single values to the variables, we make propositions, which might be true or false; and so investigation and agreement become possible, as we then have something definite to talk about.

A fundamental structural issue arises in this connection; namely, that in doing this (assigning single values to the variables) , our attitude has automatically changed to an extensional one. By using our statements with a date, we deal with definite issues, on record, which we can study, analyse, evaluate. , and so we make our statements of an extensional character, with all cards on the table, so to say, at a given date. Under such extensional and limited conditions, our statements then become, eventually, propositions, and, therefore, true or false, depending on the amount of information the maker of the statements possesses. We see that this criterion, though difficult, is feasible, and makes agreement possible.

A structural remark concerning the *A*-system may not be amiss here. In the *A*-system the ‘universal’ proposition (which is usually a propositional function) always implies *existence*. In *A* ‘logic’, when it is said that ‘all A’s are B’, it is assumed that there are A’s. It is obvious that always assuming existence leaves no place for non-existence; and this is why the old statements were supposed to be true or false. In practical life, collections of noises (spell-marks) which look like words, but which are not, are often not suspected of being meaningless, and action based on them may consequently entail unexplicable disaster. In our lives, most of our miseries do not originate in the field where the terms ‘true’ and ‘false’ apply, but in the field where they *do not apply;* namely, in the immense region of propositional functions and meaninglessness, where agreement must fail.

Besides, this sweeping and unjustified structural assumption makes the *A*-system *less general.* To the statement, ‘all A’s are B.’ the mathematician adds ‘there may or may not be A’s’. This is obviously *more general.* The old pair of opposites, true and false, may be enlarged to three possibilities—statements which might be true, or false, and verbal forms which have the appearance of being statements and yet have no meaning, since the noises used were spell-marks, *not symbols* for anything with actual or ‘logical’ existence.

Again a $\bar{A}$-system shares with the $\bar{E}$ and $\bar{N}$ systems a useful and important methodological and structural innovation; namely, it limits the validity of its statements, with weighty semantic beneficial consequences, as it tends from the beginning to eliminate undue, and often intense, dogmatism, categorism, and absolutism. This, on a printed page, perhaps, looks rather unimportant, but when *applied,* it leads to a fundamental and structurally beneficial alteration in our semantic *attitudes* and behaviour.

In the present work, each statement is merely the best the author can make in 1933. Each statement is given *definitely,* but with the semantic *limitation* that it is based on the information available to the author in 1933. The author has spared no labour in endeavouring to ascertain the state of knowledge as it exists in the fields from which his material is drawn. Some of this information may be incorrect, or wrongly interpreted. Such errors will come to light and be corrected as the years proceed.

A great source of difficulty and of possible objections is that science is, at present, so specialized that it is impossible for one man to know all fields, and that, therefore, the use of a term such as ‘science 1933’, might be fundamentally unsound. This objection should not be lightly dismissed, as it is serious. Yet it can, I believe, be answered satisfactorily. At this early stage of our enquiry, a large number of the facts of knowledge does not affect my investigation; therefore, it has not proved impossible to keep sufficiently well informed on the points which are covered. Also, the further scientific theories advance, the simpler they become. For instance, books on physics are simpler and less voluminous now than twenty years ago. Something similar could be said about mathematics. The general outlook is simpler.

The main interest of the author at this stage of his work is structural and semantic, rather than technical, and so he has only had to know enough of the technique of different sciences to be able to understand sufficiently their *structure* and *method.* Revolutionary structural and methodological advances are few in the history of mankind; and so it is possible, though not easy, to follow them up in 1933.

But the main point is that the affixing of the date has very far-reaching structural methodological and, therefore, psycho-logical semantic consequences. For instance, it changes propositional function into propositions, converts semantically one-valued intensional methods into ∞-valued extensional methods, introduces four-dimensional methods. , and so the 'date' method is to be recommended on these *structural and semantic grounds alone*. As it is beneficial to affix the date in 1933, we affix the date 1933, not to give the impression that from a technical point of view I am familiar with the results of all branches of science at that date, but to indicate that no advance in *structure* and *method* of 1933 has been disregarded. It will become obvious later in this book, when additional data have been taken into consideration, and a new summary and new abstractions made, that the result is a surprising simplification, which can be clearly understood by laymen as well as by scientists. With the help of the generalizations of new structure and ∞-valued semantics, it will be easier to follow the advance of science, because we shall then have a better outlook on the psycho-logics of science as-a-whole.

It will become clear, too, that to provide for a further elaboration of this work in the future, the establishment of a special branch of research in $\overline{A}$-systems must become a *group* activity; for, as I have been painfully aware, the production of even this outline of that branch of research has overstrained the powers of one man.

The most cheering part of this work is, perhaps, the practical results which this investigation has accomplished, combined with the simplicity of means employed. One of the dangers into which the reader is liable to fall is to ascribe too much generality to the work, to forget the limitations and, perhaps, one-sidedness which underlie it. The limitation and the generality of this theory lie in the fact that if we symbolize our human problems ($H=f(x_1, x_2, x_3, x_4, x_5, \ldots x_n)$) as a function of an enormous number of variables, the present theory deals only with a few of these variables, let us say x_1 (say, structure), x_2 (say, evaluation). , but these variables have been found, up to the present, in all our experience and all our equations.

A most important extension of the notion of ‘function’ and ‘propositional function’ has been further accomplished by Cassius J. Keyser, who, in 1913, in his discussion of the multiple interpretations of postulate systems, introduced the notion of the ‘doctrinal function’. Since, the doctrinal function has been discussed at length by Keyser in his *Mathematical Philosophy* and his other writings, by Carmichael[2], and others. Let us recall that a propositional function is defined as an ∞-valued statement, containing one or more variables, such that when single values are assigned to these variables the expression becomes a one-valued proposition. A manifold of interrelated propositional functions, usually called postulates, with all the consequences following from them, usually called theorems, has been termed by Keyser a *doctrinal function.* A doctrinal function, thus, has no specific content, as it deals with variables, but establishes *definite relations* between these variables. In principle, we can assign many single values to the variable terms and so generate many doctrines from *one* doctrinal function. In an ∞-valued $\bar{A}$-system which eliminates identity and is based on structure, doctrinal functions become of an extraordinary importance.

In an ∞-valued world of absolute individuals on objective levels, our statements can always be formulated in a way that makes obvious the use of ∞-valued terms (variables) and so the postulates can always be expressed by propositional function. As postulates establish relations or multi-dimensional order, a set of postulates which defines a doctrinal function gives, also *uniquely,* the *linguistic structure.* As a rule, the builders of doctrines do not start with sets of postulates which would explicitly involve variables, but they build their doctrine around some specific content or one special respective value for the variables, and so the *structure* of a doctrine, outside of some mathematical disciplines, has never been explicitly given. If we trace a given *doctrine with specific content* to its *doctrinal function without content,* but variable terms, then, only, do we obtain a set of postulates which gives us the *linguistic structure.* Briefly, to find the structure of a doctrine, we must formulate the doctrinal function of which the given doctrine is only a special interpretation. In non-mathematical disciplines, where doctrines are not traced down to a set of postulates, we have no means of knowing their structure, or whether *two different* doctrines originated from *one* doctrinal function, or from *two*. In other words, we have no simple means of ascertaining whether the two different doctrines have similar or different structure. Under aristotelianism, these differentiations were impossible, and so the problems of linguistic structure, propositional and doctrinal functions. , were neglected, except in the recent work of mathematicians. The entirely general semantic influence of these structural conditions becomes obvious when we realize that, no matter whether or not our doctrines are traced down to their doctrinal functions, our semantic processes and all ‘thinking’ follow *automatically* and, by necessity, the conscious or unconscious postulates, assumptions. ; which are given (or made conscious) *exclusively* by the doctrinal function.

The terms ‘proposition’, ‘function’, ‘propositional function’, ‘doctrinal function’. , are multiordinal, allowing many orders, and, in a given analysis, the different orders should be denoted by subscripts to allow a differentiation between them. When we deal with more complex doctrines, we find that in structures they represent higher order doctrines, or a higher whole, the constituents of which represent lower order doctrines. Similarly, with doctrinal functions, if we take any *system,* an analysis will discover that it is a whole of related doctrinal functions. As this situation is the most frequent, and as ‘thinking’, in general, represents a process of relating into higher order relational entities which are later *treated as complex wholes,* it is useful to have a term which would symbolize doctrinal functions of higher order, which are made up of doctrinal functions of lower orders. We could preserve the terminology of ‘higher’ and ‘lower’ order; but as these conditions are always found in all *systems,* it seems more expedient to call the higher body of interrelated doctrinal functions, which ultimately produce a system—a *system-function.* At present, the term ‘system function’ has been already coined by Doctor H. M. Sheffer[3]; but, to my knowledge, Sheffer uses his ‘system function’ as an equivalent for the ‘doctrinal function’ of Keyser. For the reasons given above, it seems advisable to limit the term ‘doctrinal function’ to the use as introduced by Keyser, and to enlarge the meaning of Sheffer’s term ‘system function’ to the use suggested in the present work, this natural and wider meaning to be indicated by the insertion of a hyphen.

In a $\overline{A}$-system, when we realize that we live, act. , in accordance with *non-el s.r,* always involving integrated ‘emotions’ and ‘intellect’ and, therefore, some explicit or implicit postulates which, by structural necessity, utilize variable, multiordinal and ∞-valued terms, we must recognize that *we live and act by some system-functions* which consist of doctrinal functions. The above issues are not only of an academic interest, as, without mastering all the issues emphasized in the present work, it is impossible to analyse he extremely complex difficulties in which, as a matter of fact, we are immersed.

At present, the doctrinal functions and the system-functions have not been worked out, and even in mathematics, where these notions originated, we speak too little about them. But in mathematics, as the general tendency is to bring all mathematical disciplines to a postulational base, and these postulates always involve multiordinal and ∞-valued terms, we actually produce doctrinal or system-functions, as the case may be. In this way, we find the *structure* of a given-doctrine or system, and so are able to compare the structures of different, and sometimes very complex, verbal schemes. Similar structure-finding methods must be applied some day to all other, at present, non-mathematical disciplines. The main difficulty, in the search for structure, was the absence of a clear formulation of the issues involved and the need for a $\overline{A}$-system, so as to be able to *compare two systems* the comparison of which helps further structural discovery. It is not claimed that either the A or $\overline{A}$ system-functions have been formulated here, but it seems that, in the presence or absence of identification, we find a fundamental postulate which, once formulated, suggests a comparison with experience. As we discover that ‘identity’ is invariably false to facts, this A postulate must be rejected from any future $\overline{A}$-system.

It happens that any new and revolutionary doctrine or system is always based on a new doctrinal or system-function which establishes its new structure with a new set of relations. Thus, any new doctrine or system, when traced to its postulates, allows us to verify and scrutinize the initial postulates and to find out if they correspond to experience, .

A few examples will make it clearer. Cartesian analytical geometry is based on one system-function, having one system-structure, although we may have indefinitely many different cartesian co-ordinates. The vector and the tensor systems also depend on two different system-functions, different from the cartesian; they have three different structures. Intertranslations are possible, but only when the fundamental postulates do not conflict Thus, the tensor language gives us invariant and intrinsic relations, and these can be translated into the cartesian relations. It seems certain, however, although I am not aware that this has been done, that the indefinitely many extrinsic characteristics which we can manufacture in the cartesian system, cannot be translated into the tensor language, which does not admit extrinsic characteristics.

Similar relations are found between other doctrines and systems, once their respective structural characteristics are discovered by the formulating of their respective functions, which, by the explicit or implicit postulates, determine their structure.

Thus all existing schools of psychotherapy, prior to 1933, result from *one* system-function which underlies *implicitly* the system originated by Freud.[4] The *particular* freudian doctrine is only one of the indefinitely many variants of *similar system-structure,* which can be manufactured from the one system-function underlying the particular freudian system. In other words, it is of no importance what ‘complex’ we emphasize or manufacture, the *structural principles* which underlie this new freudian and revolutionary system-function remain unchanged. From this point of view, all existing schools of psychotherapy could be called ‘cartesian’, because, although they all have *one* general system-structure, yet they allow indefinitely many particular variations. The present $\overline{A}$-system suggests that the ‘cartesian’ school of psychotherapy is still largely *A*, *el* and fundamentally of one structure.

The present system involves a different system-function of different structure, rejecting identity, discovering the ‘structural unconscious’, establishing psychophysiology, . The mutual translatability follows the rules of general semantic principles or conditions which apply also to mathematics; namely, that a $\overline{A}$-system, being based on relations; on the elimination of identity; on structure. , gives us only intrinsic characteristics and might be called the ‘tensor’ school of psychotherapy. This system allows all the intrinsic characteristics discovered, no matter by whom, but has no place for the indefinitely many, quite consistent, yet irrelevant metaphysical, extrinsic characteristics, which we can manufacture at will.

Without the realization of the structural foundations emphasized in the present system, it is practically impossible not to confuse linguistic structural issues, which lead inevitably to semantic blockages. When we deal with doctrines or systems of *different* structure, each of which involves different doctrinal or system-functions, it is of the utmost importance to keep them at first *strictly separated;* to work out each system by itself, and only after this is accomplished can we carry out an independent investigation as to the ways they *mutually intertranslate.* Let me again repeat, that the mixing of different languages of different structures is fatal for clear ‘thinking’. Only when a system is traced to its system-function, and the many implications worked out in their *un-mixed* form, can we make a further *independent* investigation of the ways in which the different systems intertranslate. As a general rule, every new scientific system eliminates a great deal of spurious metaphysics from the older systems. In practice, the issues are extremely simple if one decides to follow the general rule; namely, either completely to reject or completely to accept *provisionally,* at a given date, a new system; use *exclusively* the structurally new terms; perform our semantic operations exclusively in these terms; compare the conclusions with experience; perform *new* experiments which the structurally new terminology suggests; and only then, as an independent enquiry, investigate how one system translates into the other. In those translations, which correspond to the transformation of frames of reference in mathematics, we find the most important invariant characteristics or relations which survive this translation. If a characteristic appears in all formulations, it is a sign that this characteristic is intrinsic, belongs to the subject of our analysis, and is not accidental and irrelevant, belonging only to the accidental structure the language we use. Once these invariant, intrinsic characteristics are discovered, and there is no way to discover them except by reformulating the problems in different languages (in mathematics we speak about the transformation of frames of reference), we then know that we have discovered invariant relations, which survive transformation of different forms of representations, and so realize that we are dealing with something genuinely important, *independent* from the structure of the language we use.

History shows that the discovery of isolated, though interesting, facts has had less influence on the progress of science than the discovery of *new system-functions* which produce *new* linguistic *structures and new methods.* In our own lifetime, some of the most revolutionary of these advances in structural adjustment and method have been accomplished. The work of Einstein, the revision of mathematical foundations, the new quantum mechanics, colloidal science, and advances in psychiatry, are perhaps structurally and semantically the most important. There seems no escape from admitting that no modern man can be really intelligent in 1933 if he knows nothing about these structural scientific revolutions. It is true that, because these advances are so recent, they are still represented in very technical terms; their system-functions have not been formulated, and so the deeper structural, epistemological and semantic simple aspects have not been worked out. These aspects are of enormous human importance. But they must be represented without such an abundance of dry technicalities, which are only a means, and not an end, in search for structure.

A scientist may be very much up to date in his line of work, let us say, in biology; but his physico-mathematical structural knowledge may be somewhere in the eighteenth or nineteenth century and his epistemology, metaphysics, and structure of language of 300 B.C. This classification by years gives a fairly good picture of his semantic status. Indeed, we can foretell quite often what kind of reaction such a man will exhibit.

This functional, propositional-function, and system-function structural attitude is in accord with the methods developed by psychiatry. In psychiatry, 'mental' phenomena are considered, in some instances, from the point of view of arrested development; in others, as regression to older and more primitive levels. With this attitude and understanding, we cannot ignore this peculiar intermixing of different personalities in one man when different aspects of him exhibit *s.r* of different ages and epochs of the development of mankind. In this connection should be mentioned the problem of the multiple personalities which often occur in the 'mentally' ill. Such splitting of personality is invariably a serious semantic symptom, and a person who exhibits different ages in his semantic development, as, for instance, 1933 in some respects, sixteenth century in others, and 300, or even 5000, B.C. in still others, cannot be a well co-ordinated individual. If we teach our children, whose nervous systems are *not* physically finished at birth, doctrines structurally belonging to entirely different epochs of human development, we ought not to wonder that semantic harm is done. Our efforts should be to co-ordinate and integrate the individual, help the *nervous* system, and not split the individual semantically and so disorganize the nervous system.

It is necessary to remember that the organism works as-a-whole. In the old days we had a comforting delusion that science was a purely 'intellectual' affair. This was an *el* creed which was structurally false to facts. It would probably be below the dignity of an older mathematician to analyse the 'emotional' values of some piece of mathematical work, as, for instance, of the 'propositional function'. But such a mathematician probably never heard of psychogalvanic experiments, and how his 'emotional curve' becomes expressive when he is solving some mathematical problem.

In 1933, we are not allowed to follow the older, seemingly easier, and simpler paths. In our discussion, we have tried to analyse the problems at hand as ∞-valued manifestations of human behaviour. We were analysing the doings of Smith, Brown. , and the semantic components which enter into these forms of behaviour must be especially emphasized, emphasized because they were neglected. In well-balanced persons, all psycho-logical aspects should be represented and should work harmoniously. In a theory of sanity, this semantic balance and co-ordination should be our first aim, and we should, therefore, take particular care of the neglected aspects. The *non-el* point of view makes us postulate a permanent connection and interdependence between all psycho-logical aspects. Most human difficulties, and ‘mental’ ills, are of *non-el* affective origin, extremely difficult to control or regulate by *el* means. Yet, we now see that purely technical scientific discoveries, because structural, have unsuspected and far-reaching beneficial *affective* semantic components. Perhaps, instead of keeping such discoveries for the few ‘highbrows’, who never use them fully, we could introduce them as structural, semantic and *linguistic* devices into elementary schools, with highly beneficial psycho-logical results. There is really no difficulty in explaining what has been said here about structure to children and training them in appropriate *s.r*. The effect of doing so, on sanity, would be profound and lasting.